

Manuscript on fine structure, inner model
theory, and the core model below one
Woodin cardinal

Ronald B. Jensen

Preface

Here are the first four chapters of a prospective book. It is intended to provide a detailed introduction to fine structure theory, ultimately leading up to a proof of the Covering Lemma for the Core Model under the assumption that there is no inner model with a Woodin cardinal.

I am grateful to various colleagues who have helped and encouraged me in this project. I especially want to thank Ralf Schindler, who has provided steadfast support at every stage. I am completely dependent on technical typing and, therefore, owe a special debt to Ms. Martina Pfeiffer, who typed the initial sections, and to Dr. Fabiana Castiblanco, who very competently continued this work.

A group of “anonymous donors”, unknown to me, is helping to defray the cost of producing the manuscript. For this I am exceedingly grateful. Last but not least I thank my wife Hilde, who gives me the strength to continue.

Ronald Jensen

Contents

0	Preliminaries	1
1	Transfinite Recursion Theory	9
1.1	Admissibility	9
1.1.1	Introduction	9
1.1.2	Properties of admissible structures	11
1.1.3	The constructible hierarchy	19
1.2	Primitive Recursive Set Functions	22
1.2.1	PR Functions	22
1.2.2	PR Definitions	30
1.3	Ill founded ZF^- models	33
1.4	Barwise Theory	35
1.4.1	Syntax	35
1.4.2	Models	41
1.4.3	Applications	43
2	Basic Fine Structure Theory	49
2.1	Introduction	49
2.2	Rudimentary Functions	51

2.2.1	Condensation	67
2.3	The J_α hierarchy	68
2.3.1	The J_α^A -hierarchy	74
2.4	J -models	77
2.5	The Σ_1 projectum	87
2.5.1	Acceptability	87
2.5.2	The projectum	89
2.5.3	Soundness and iterated projecta	96
2.6	Σ^* -theory	102
2.7	Liftups	129
2.7.1	The Σ_0 liftup	129
2.7.2	The $\Sigma_0^{(n)}$ liftup	135
3	Mice	155
3.1	Introduction	155
3.2	Extenders	159
3.2.1	Extendability	167
3.2.2	Fine Structural Extensions	169
3.2.3	n -extendibility	174
3.2.4	$*$ -extendibility	178
3.2.5	Good Parameters	181
3.3	Premice	186
3.4	Iterating premice	205
3.4.1	Introduction	205
3.4.2	Normal iteration	208

3.4.3	Padded iterations	215
3.4.4	n -iteration	217
3.4.5	Copying an iteration	219
3.4.6	Copying an n -iteration	224
3.5	Iterability	225
3.5.1	Normal iterability	225
3.5.2	The comparison iteration	227
3.5.3	n -normaliterability	230
3.5.4	Iteration strategy and copying	231
3.5.5	Full iterability	231
3.5.6	The Dodd–Jensen Lemma	234
3.5.7	Copying a full iteration	236
3.5.8	The Neeman–Steel lemma	237
3.5.9	Smooth iterability	241
3.5.10	n -full iterability	241
3.6	Verifying full iterability	244
3.6.1	Introduction	244
3.6.2	Pseudo projecta	244
3.6.3	Mirrors	264
3.6.4	The conclusion	280
3.7	Smooth Iterability	286
3.7.1	Insertions	286
3.7.2	Reiterations	307
3.7.3	A first conclusion	332

3.7.4	Reiteration and Inflation	337
3.7.5	Smooth Reiterability	354
3.7.6	The final conclusion	362
3.8	Unique Iterability	371
3.8.1	One small mice	371
3.8.2	Woodiness and non unique branches	373
3.8.3	One smallness and unique branches	387
3.8.4	The Bicephalus	394
4	Properties of Mice	399
4.1	Solidity	399
4.2	Phalanx Iteration	410
4.3	Solidity and Condensation	441
4.3.1	Solidity	442
4.3.2	Soundness and Cores	448
4.3.3	Condensation	452
4.4	Mouselikeness	461
4.4.1	Σ_1 -acceptability	474
4.4.2	Mouselikeness in 1-small premice	478
5	The Model K^c	481
5.1	Introduction	481
5.2	The Steel Array	487
5.3	Robust Premice	499
5.3.1	The Chang hierarchy	499

5.3.2	Robustness	501
5.4	Worlds	507
5.4.1	Good Worlds	511
5.4.2	The Relation R	512
5.5	Enlargements	520
5.6	The Bicephalus	552
5.7	The model K^c	558

