Midterm II Practice problems

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1. Let k be an algebraically closed field of characteristic $\neq 2$. Determine the fibers of the morphism

$$f : X = \operatorname{Spec} k[x_1, x_2, x_3]/(x_1^2 - x_2^2 + x_3^2 - 1) \to \operatorname{Spec} k[x_3].$$

Which fibers are reduced, which are irreducible?
Does your answer change if $\operatorname{char} k = 2$?

2. What are the fibers of $f : \operatorname{Spec} \mathbb{R}[y] \to \operatorname{Spec} \mathbb{R}[x]$ induced by the ring homomorphism $x \mapsto y^2$? Which fibers are reduced, which irreducible?

3. Let A be a local ring. Show that $\operatorname{Spec}(A)$ is connected.

4. Is an open subset of a locally ringed space a locally ringed space?

5. Find all the zero-dimensional closed subschemes of $\mathbb{C}[x, y]/(x^2 - y^3)$ of length $0, 1, 2, 3$. You may assume that the underlying reduced subscheme is the origin (why?).

6. Let $f : X \to X$ be the identity morphism. What is the diagonal morphism associated to f? Is f separated?

7. Let X be a separated scheme over the affine scheme $S = \operatorname{Spec} A$. Then the intersection of any two open affine subschemes of X is an affine open subset of X.

8. A scheme X and its reduction X_{red} have the same dimension.

9. Show that a morphism of schemes $f : X \to S$ is separated if and only if for every scheme S_0 over S every section of the base change $f_0 : X \times_S S_0 \to S_0$ is a closed immersion.

10. Let X be a scheme of finite type over a field. Show that the set of closed points of X is dense in X.

11. Hartshorne II.3.11 part d)