General information

The exam will be an oral exam lasting twenty minutes. The schedule will be posted on the course website.

At least fifty percent of the exam questions will be taken from the list below. This list is intended to facilitate your review process and to give an idea of the types of questions I will ask. However, this list is not necessarily exhaustive: any material which was covered in class and which was not explicitly announced as being non-examinable is fair game.

- Define the following notions:
  1. symplectic vector space
  2. isotropic, coisotropic, symplectic, and Lagrangian subspaces
  3. symplectic complement
  4. symplectic manifold
  5. symplectomorphism
  6. the canonical 1-form on T^*L (be prepared to prove that its exterior)
- Give five examples of symplectic manifolds. (Each of which should be distinct in a non-silly way, i.e. you can't take $(\mathbb{R}^{2n}, \omega_0)$ for $n = 1, 2 \ldots, 5$.)
- Which spheres admit a symplectic structure? Why?
- What is the statement of the Darboux theorem?
- What is a Hamiltonian? Prove that the time-1 flow of a (possibly time-dependent) Hamiltonian is a symplectic isomorphism (also called symplectomorphism).
- Give an example of a Hamiltonian vector field on a surface. Draw some arrows in this vector field.
- Let (M, ω) be a connected symplectic manifold. Prove that the group of symplectic isomorphisms acts transitively on points (i.e. you can send any point to any other point).
- What is a symplectic isotopy? What is a Hamiltonian isotopy? Give an example of a symplectic isotopy which is not Hamiltonian.

- Define the Poisson bracket of two functions on a symplectic manifold. If the Poisson bracket of F and G vanishes, what can we say about the behavior of F on the orbits of the flow of X_G?
- What is the relationship between the Poisson bracket of F and G and the Lie bracket of the corresponding Hamiltonian vector fields?
- State Moser's trick. Give an example of an important consequence.
- State the Euler-Lagrange equations and Hamilton's equations. What transformation relates these two systems?
- State the nearby Lagrangian conjecture. Prove it for T^*S^1. (Bonus: for what other manifolds is is known?)
- what is a symplectic toric manifold? Examples?
- State Delzant's theorem.
- Define what is a Lagrangian submanifold of a symplectic manifold.
- Prove that every symplectic manifold contains a closed Lagrangian submanifold. (Hint: Darboux's theorem)
- State the Arnold conjecture (Hamiltonian and Lagrangian version). Explain why the former follows from the later (Hint: consider the graph)
- State the Gromov–Eliashberg theorem?
- State the Gromov non-squeezing theorem.
- What is a symplectic capacity?
- What is the Hofer–Zehnder capacity? Say a few words about the proof.
- Define the Hofer metric on $Ham(M)$. Discuss how it is related to the C^0 metric.