

# Homological projective duality HPD<sup>1</sup>

Summer term 2020, Tuesday 2-4pm, SR 0.011/Zoom

The seminar is devoted to the concept of HPD introduced by Alexander Kuznetsov some 15 years ago. The emphasis will be on examples, we want to see HPD in action. We will follow the survey article [T] to which we also refer for most of the references. The paper is deceptively short concerning some of the examples and going back to the original sources is advised.

This is a first draft of the program. We should feel free to change it if during the seminar we realize that we need more time for certain topics or get interested in other applications of HPD than the ones planned for at the moment. Please get in touch with me (**email:** huybrech@...) if you have comments or questions on the talks. Concerning the distributions of the talks: first come first serve.

## 1. Classical semi-orthogonal decompositions (28 April, Denis Nesterov)

Recall the notions of exceptional collections and semi-orthogonal decompositions. Discuss the the following four examples: (i) Beilinson spectral sequence; (ii) SOD for projective bundles; (iii) SOD for blow-ups; (iv) SOD for Brauer–Severi varieties.

## 2. HPD I (5 May, Yajnaseni Dutta)

Cover Section 3 in [T]. Proposition 3.6 and its proof should be presented in detail.

## 3. Lefschetz decompositions & HPD II (12 May, Georg Oberdieck)

Before talking about HPD, one always needs to find a Lefschetz decomposition. Follow Section 4.1 and add the example of Grassmannians [F]. Introduce Theorem 4.7 in [T] (HPD II) and explain how to think about it. At this point, we do not enter its proof and may not even discuss the notion of left mutation which are the key technique. This will be done later (or not at all). It seems more interesting to see how the theorem works in examples.

## 4. & 5. Quadrics and spinor bundles (19 May & 26 May, Miro Mauri, Thorsten Beckmann)

These two talks (with two speakers!) should cover Sections 5.1 and 5.2 in [T]. The literature on quadrics is vast and you will have to pick the things you find interesting. In addition to the references in [T], in particular [ABB] there should be looked at, the paper [A] could be helpful.

## 6. HPD for quadrics (9 June, Daniel Huybrechts)

This is closely related to the previous two talks and should survey the more recent paper [KP1].

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<sup>1</sup>The seminar is in principle suitable for master students with a solid background in algebraic geometry and some knowledge in derived categories. Should you need some refresher for the latter, we will provide an additional tutorial before the start of the seminar if needed.

### **7. Pfaffian examples** (16 June, Pieter Belmans)

Discuss the following two examples mentioned in Section 5.3 in [T]: Borisov–Caldararu and quintic threefold. (The paper [RS] would lead us to far astray. But of course if someone feels inspired, we could also talk about that one.)

### **8. Cubics** (23 June, Alberto Cattaneo)

The talk should discuss the three examples of HPD related to cubic fourfolds mentioned in Section 5 of [T]: Pfaffians, nodal cubics, and cubics containing a plane. The emphasis should be put on the geometry of the situations and not on the other two topics mentioned there, rationality and Hassett vs Kuznetsov. (We could also decide not to talk about cubics at all, if we feel that we know this already sufficiently well.)

### **9. Left mutations or joints??** (30 June, ??)

At this point (if not totally exhausted already), we will have a choice: We could either go into the two papers [KP2] and [KP3] or come back to left mutations and the proof of HPD II. (My preference would be for the former.)

### **11. Cayley trick** (??)

This should elaborate the content of the ‘Digression’ in Section 3 in [T]. The topic is somewhat optional. We may also do it later or drop it altogether.

## **References**

[A] N. Addington *Spinor sheaves on singular quadrics* arXiv:0904.1766

[Fo] A. Fonarev *On minimal Lefschetz decompositions for Grassmannians* arXiv:1108.2292

[KP1] A. Kuznetsov, A. Perry *Homological projective duality for quadrics* arXiv:1902.09832

[KP2] A. Kuznetsov, A. Perry, *Categorical cones and quadratic homological projective duality* arXiv:1902.09824

[KP3] A. Kuznetsov, A. Perry *Categorical joins* arXiv:1804.00144

[T] R. Thomas *Notes on homological projective duality* arXiv:1512.08985