

# Moduli spaces of K3 surfaces and hyperkähler manifolds

Bonn–Paris, Fall 2020

Start: November 6, Time: Friday 10:15-11:45 via Zoom.

The following is a list of possible topics for the reading seminar. It is a little difficult to predict how much time is necessary to say something meaningful about the various results and proofs. Some of the results can be explained in full detail, others will be beyond the scope of the seminar. The hope is that we can join the different expertise of the participants to get a global picture. All dates are provisional. Should we need more or less time for one of the topics or swap talks we should feel free to do so.

**Noether–Lefschetz divisors** (Nov 6 & 13: Fabrizio Anella, Olivier Debarre, Emre Sertoz)  
We should start with a general introduction into K3 moduli spaces and special cycles. Then, NL divisors and higher codimension NL loci [MP13], [BLMM17, Sec. 1.5] should be introduced as well as the subring  $NL^* \subset CH^*(\mathcal{M}_d)$  generated by them. State [BLMM17, Thm. 1.2] that proves the conjecture in [MP13] that  $H^2(\mathcal{M}_d, \mathbb{Q})$  is generated by NL classes. Depending on how fast we want to go, this may be too much for one talk.

**Hodge line bundle** (Nov 20: Daniel Huybrechts)

The goal here is to prove the main result in [vdGK05] saying that  $\mathbb{Q}[v]/(v^{18})$  injects into the cohomology ring of the moduli space of quasi-polarized varieties. The result has been extended in [Pe19] which should also be covered. Also explain how to view the Hodge line bundle as a class in the NL subring [Ma14, MP13].

**Tautological rings** (Nov 27: Yagna Dutta)

The aim of this talk is to introduce the (strict) tautological ring of the moduli space. Follow [MOP17, Sec. 4] or [PY20, Sec. 1]. Introduce  $\kappa$ -classes, discuss some of the explicit examples and state the conjecture predicting that all tautological classes are contained in the NL subring [MOP17, Sec. 4.5].

**Moduli spaces of hyperkähler manifolds.** (Dec 4: Ignacio Barros & Emanuele Macrì)

This is a preparation for the next talk and partially a repetition of standard facts for K3 surfaces now in the setting of hyperkähler manifolds. Follow [BL19, Sec. 2] or some of the other standard sources. Sketch the proof for the existence of a universal family over the moduli space of marked hyperkähler manifolds (easy for K3, a little surprising for HK) in [Ma17].

**Tautological ring in cohomology** (Dec 11: Mirko Mauri)

Sketch the arguments in [BL19] proving that the Noether–Lefschetz ring equals the tautological ring in cohomology. There is also an interesting application to the cohomological Franchetta conjecture.

**Non-NL divisors** (Dec 18: Claire Voisin)

Not all divisors in HK moduli spaces are NL. One example was studied in [RV17] and made more explicit in [AA18]. Are there others?

**Tautological ring in Chow** (Jan 8 & 15: Georg Oberdieck & Johannes Schmitt)

The full conjecture in the Chow ring has been proved in [PY20] by means of GW theory. This talk should explain the main ideas. It probably requires two talks and one needs to explain the necessary background on GW theory.

### Further possible topics:

- Moduli spaces in positive characteristic, e.g. vdGeer–Katsura: <https://arxiv.org/pdf/math/9910061.pdf> and vdGeer: <https://arxiv.org/pdf/1511.00345.pdf>
- Unboundedness of Picard number  $\rho(\mathcal{M}_d)$ : O’Grady: <https://projecteuclid.org/euclid.dmj/1077304882>
- Maulik–Pandharipande conjecture for low genus: Greer, Li, Tian <https://arxiv.org/pdf/1402.2330.pdf>
- Compactifications of moduli spaces of K3 surfaces (work by Alexeev et al and others).

## References

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- [PY20] R. Pandharipande, Q. Yin *Relations in the tautological ring of the moduli space of K3 surfaces*. JEMS 22 (2020), 213–252.
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