University Bonn — WS 2017/18 Ursula Hamenstädt

Advanced Geometry I

Exercises 2 17. October 2017

2.1. (a) Let $f : U \subset \mathbb{C}^n \to \mathbb{C}$ be a holomorphic function. Show that the differential df of f, viewed as a $\mathbb{C}$-valued one-form on $\mathbb{C}^n$, is of the form

$$df = \sum_{i=1}^{n} a_i dz_i$$

with holomorphic functions a_i.

(b) Let $f : U \subset \mathbb{C}^n \to \mathbb{C}$ be a continuously differentiable map. Show that f is holomorphic if

$$df = \sum_{i=1}^{n} a_i dz_i$$

for some continuous functions a_i.

2.2. (a) Show that the complex projective line $\mathbb{C}P^1$ is biholomorphic to the sphere $\mathbb{C} \cup \{\infty\}$.

(b) (harder) Let $1, i$ be the standard basis of $\mathbb{C}$ as a vector space over $\mathbb{R}$. Define $\Lambda_1 = \{k + i\ell \mid k, \ell \in \mathbb{Z}\}$ and $\Lambda_2 = \{k + e^{\pi i/\sqrt{2}}\ell \mid k, \ell \in \mathbb{Z}\}$. Are the two-tori $T_1 = \mathbb{C}/\Lambda_1$ and $T_2 = \mathbb{C}/\Lambda_2$ biholomorphic? Justify your answer.

2.3. Let M be a complex manifold, with universal covering $\tilde{M}$. Show that the deck group acts as a group of biholomorphic transformations on $\tilde{M}$.

2.4. Show that any invertible linear map $A \in GL(n+1, \mathbb{C})$ acts as a biholomorphic transformation on $\mathbb{C}P^n$.