University Bonn — SS 2017	Ursula Hamenstädt
Advanced Geometry II	
Exercises 9	21. June 2017

9.1. (a) Show that the free group with two generators is a hyperbolic group.

(b) Show that the Gromov boundary of the free group with two generators is a Cantor set (=compact, totally disconnected, no isolated points).

(c) Let P be a hyperbolic pair of pants with geodesic boundary. Show that P is the convex core of the quotient of $\mathbb{H}^3$ by a convex cocompact Kleinian group Γ whose limit set is a Cantor set.

(d) Let $\mathrm{Hull}(\Lambda(\Gamma))$ be the convex hull of the limit set of the group Γ as in (c). Show that through every point of $\mathrm{Hull}(\Lambda(\Gamma))$ there passes a geodesic ray which is entirely contained in $\mathrm{Hull}(\Lambda(\Gamma))$. Is there also a geodesic line?

9.2. Let Γ be a convex cocompact Kleinian group, with limit set $\Lambda \neq S^2$ and convex hull $\mathrm{Hull}(\Lambda)$

(a) Show that $\mathrm{Hull}(\Lambda) = \cap H$ where H runs through all closed half-spaces whose closure in $\mathbb{H}^3 \cup S^2$ contains Λ.

(b) Show that for every point $x \in \partial\mathrm{Hull}(\Lambda) \subset \mathbb{H}^3$ (ie the boundary of $\mathrm{Hull}(\Lambda)$) there exists a half-space H whose closure contains Λ and so that $x \in \partial H$. The boundary of such a half-space is called a *supporting hyperplane* for $\partial\mathrm{Hull}(\Lambda)$.

(c) Show that the closure in $\mathbb{H}^3 \cup S^2$ of any supporting hyperplane contains a point in Λ.

(d) Deduce from (c) that through any point $x \in \partial\mathrm{Hull}(\Lambda)$ passes a geodesic ray entirely contained in $\mathrm{Hull}(\Lambda)$.

9.3. Let $\Gamma < PSL(2,\mathbb{R})$ be the fundamental group of a closed hyperbolid surface S and assume that Γ_θ is obtained from $\Gamma = \pi_1(S)$ by bending (θ is sufficiently small) at a geodesic γ. Let Λ_θ be the limit set of Λ_θ. Show that the Γ_θ-invariant pleated surface H_θ is contained in $\mathrm{Hull}(\Lambda_\theta)$.

9.4. Notations as in Exercise 9.3. Show that the limit set Λ_θ depends continuously on θ in the following sense. Fix a totally geodesic hyperbolic surface Y with geodesic boundary (the universal covering of a component of $S - \gamma$) and normalize so that $Y \subset H_\theta$ for all θ. Let $x \in Y$ be a fixed point and for each θ let β_θ be a geodesic ray issuing from x which is contained in $\mathrm{Hull}(\Lambda_\theta)$. Then any accumulation point of β_θ in the space of geodesic rays from x is contained in $\mathrm{Hull}(\Lambda) \sim \mathbb{H}^2$.