University Bonn — SS 2017	Ursula Hamenstädt
Advanced Geometry II	
Exercises 7	31. May 2017

7.1. Let S be a complete hyperbolic surface which is obtained from a compact surface Σ with one boundary geodesic by glueing a hyperbolic flaring cylinder to the boundary. Let $\Gamma < PSL(2,\mathbb{R}) \subset PSL(2,\mathbb{C})$ be the fundamental group of S. Show that the domain of discontinuity $\Omega(\Gamma)$ of Γ is connected.

7.2. Let γ be a separating closed geodesic on a closed hyperbolic surface S, $\Gamma_\theta < PSL(2,\mathbb{C})$ be the group obtained by bending S along γ with angle θ (notation as in the lecture). (We assume that everything is normalized in such a way that each of the groups Γ_θ contains $\pi_1(S_1)$ for a component S_1 of $S - \gamma$ and fixed basepoint. We furthermore assume that θ is small enough). Show that the limit set $\Lambda(\Gamma_\theta)$ of Γ_θ depends continuously on θ in the following sense. Assume that $\theta_i \to \theta$.

(i) If $x_i \in \Lambda(\Gamma_{\theta_i})$ for all i, then any accumulation point of the sequence (x_i) is contained in $\Lambda(\Gamma_\theta)$.

(ii) If $x \in \Lambda(\Gamma_\theta)$ then there exists a sequence $x_i \in \Lambda(\Gamma_i)$ with $x_i \to x$.

7.3. Notations as in the previous exercise. Let $\Lambda(\Gamma_\theta)$ be the limit set of Γ_θ; then any geodesic connecting two distinct points in $\Lambda(\Gamma_\theta)$ is contained in a uniformly bounded neighborhood of the Γ_θ-invariant pleated surface H_θ defined by Γ_θ as in the lecture.

7.4. Let S be a closed hyperbolic surface, $\Gamma = \pi_1(S) < PSL(2,\mathbb{R})$. Show that there exists $L > 1, C > 0$ and for each $\xi \in \partial\mathbb{H}^2$, $x \in \mathbb{H}^2$ there exists an (L,C)-quasi-geodesic $\zeta : [0,\infty) \to \mathbb{H}^2$ starting at $\zeta(0) = x$ such that $\zeta(t) \to \xi$ $(t \to \infty)$ and $\zeta(t) \in \Gamma x$ for all t.