University Bonn — SS 2017	Ursula Hamenstädt
Advanced Geometry II	
Exercises 6	24. May 2017

6.1. Call an isometry $A \in SO(3,1)^+ = PSL(2,\mathbb{C})$ *elliptic* if it is not the identity and fixes at least one point in $\mathbb{H}^3$.

(a) Show that an elliptic isometry fixes at least two points in $\partial\mathbb{H}^3 = S^2$.

(b) Show that an isometry $A \in PSL(2,\mathbb{C})$ is elliptic if and only if $|\mathrm{tr}(A)| < 2$.

6.2. Call an isometry $A \in PSL(2,\mathbb{C})$ *hyperbolic* if $|\mathrm{tr}(A)| > 2$. Show that the following are equivalent.

(i) A is hyperbolic.

(ii) A preserves a unique geodesic $\tilde{\gamma} \subset \mathbb{H}^3$ and acts on it as a translation.

(ii) A preserves some biinfinite quasi-geodesic and acts on it as a translation.

(vi) The infimum $\delta(A) = \inf\{d(x, Ax) \mid x \in \mathbb{H}^3\}$ is assumed, and it is positive,

6.3. Let S be a closed hyperbolic surface and let γ be a simple closed separating geodesic on S as in the lecture. Let θ be sufficiently small that the bending Γ_θ of the group $\Gamma = \pi_1(S) \subset PS(2,\mathbb{C})$ at γ with bending angle θ is discrete.

(a) Show that any element $g \in \Gamma_\theta$ can be represented in $M_\theta = \Gamma_\theta\backslash\mathbb{H}^3$ by a closed geodesic whose length does not exceed the length of the corresponding closed geodesic in S.

(b) Show that for fixed $g \in \Gamma$, the length of the corresponding geodesic in M_θ as in (b) depends continuously on θ.

(c) Characterize those elements whose length in M_θ coincides with their length in S.

6.4. Show that for every $\theta > 0$ there exists a piecewise geodesic path $\alpha : \mathbb{R} \to \mathbb{H}^2$ which consists of geodesic segments meeting with a breaking angle smaller than θ which is not a quasi-geodesic.

The student council of mathematics will organize the math party on 01/06 in N8schicht. The presale will be held on Mon 29/05, Tue 30/05 and Wed 31/05 in the mensa Poppelsdorf. Further information can be found at fsmath.uni-bonn.de ¡http://fsmath.uni-bonn.de/¿