University Bonn — SS 2017	Ursula Hamenstädt
Advanced Geometry II	
Exercises 4	9. May 2017

4.1. Let $A(\ell, R)$ be a standard annulus of width ℓ and height R, i.e. the closed tubular neighborhood of radius R about a closed geodesic of length ℓ in a complete hyperbolic cylinder C. Show.

(i) $A(\ell, R)$ is a submanifold of C with boundary.

(ii) $A(\ell, R) \subset C$ is *locally convex*. This means that if $x, y \in A(\ell, R)$ then a shortest geodesic connecting x to y in C is contained in $A(\ell, R)$ as well.

4.2. Use the collar theorem to show: There exists a constant $c > 0$ with the following property. Let S be a closed hyperbolic surface and let $\alpha \subset S$ be a closed non-contractible loop of length at most c. Then the set of all points $x \in S$ with the property that there exists a closed loop of length at most c which is freely homotopic to α and of length at most c is arcwise connected (and in fact homeomorphic to an annulus).

4.3. Use the collar theorem and Problem 4.2 to show that there is a number $b > 0$ with the following property. Let γ be a closed geodesic of length $\ell \leq c$ with c as in 4.2 above. Let $R_0 = \sup\{R > 0 \mid \text{ the standard annulus of radius } R \text{ about } \gamma \text{ is embedded}\}$. Then the injectivity radius of S at any point p in the boundary the annulus of radius R_0 about γ is at least b.

4.4. Let γ be a closed geodesic in a closed hyperbolic surface S of length $\ell > 0$. Show: There is a standard complete hyperbolic cylinder C with fundamental group $\langle[\gamma]\rangle$ (generated by the class of γ for a basepoint on γ) which covers S. There is a covering projection $C \to S$ which maps the core geodesic of C onto γ. Furthermore, if $x, y \in C$ are mapped to the same point in S then any arc in C connecting x to y maps in S to a loop which is not homotopic to zero.