University Bonn — SS 2017 Ursula Hamenstädt

Advanced Geometry II

Exercises 10 28. June 2017

10.1. Let $x \in \mathbb{H}^3$ be an arbitrary point, let $v \in T_x^1\mathbb{H}^3$ and let $\alpha \in (0, \pi/2)$.

(a) Show that the cone $C(v, \alpha) = \{\exp(tw) \mid t \geq 0, \langle v, w\rangle \leq \alpha\}$ is convex.

(b) Show that for any other $\beta \in (0, \pi/2)$, the cones $C(v, \alpha), C(v, \beta)$ are quasi-isometric.

(c) Now let $D \subset \mathbb{H}^2 \subset \mathbb{H}^3$ be a closed disk of radius $r > 0$ and let N be the oriented normal field of $\mathbb{H}^2$. Show that the cone $C(v, \alpha)$ is quasi-isometric to $\{\exp(tN(y)) \mid t \geq 0, y \in D\}$.

10.2. Let Γ be a convex cocompact Kleinian group, with non-empty domain of discontinuity $\Omega(\Gamma) = S^2 - \Lambda(\Gamma)$, and let $M = \Gamma\backslash\mathbb{H}^3$.

(a) Show that there exists a sequence of points $(x_i) \subset M$ such that the pointed hyperbolic manifolds (M, x_i) converge in the geometric topology to $\mathbb{H}^3$.

(b) Show that every sequence $(x_i) \subset \mathrm{Core}(M)$ has a subsequence so that (M, x_i) converges in the geometric topology to M.

10.3. Let $\Gamma < PSL(2, \mathbb{R})$ be the fundamental group of a closed hyperbolid surface S and assume that Γ_θ is obtained from $\Gamma = \pi_1(S)$ by bending (θ is sufficiently small) at a geodesic γ. Let $\theta_i \to 0$; show that there exists a sequence $y_i \in \Gamma_{\theta_i}\backslash\mathbb{H}^3 = M_{\theta_i}$ so that the pointed manifolds (M_{θ_i}, y_i) converge in the geometric topology to (M, y) where $M = \Gamma\backslash\mathbb{H}^3$.

10.4. (Harder) Let (M_i) be a sequence of quasi-fuchsian manifolds whose injectivity radius is bounded from below by some $\delta > 0$. Show that there exists a sequence of points $y_i \in M_i$ so that the sequence of pointed manifolds (M_i, y_i) does not have a subsequence which converges in the geometric topology to $\mathbb{H}^3$.