1 RHA, SS 13, Exercise Sheet 5

Due May 15. 2013.

Exercise 1:

Carefully write down the bushes argument to prove that a compact set in $\mathbf{R}^n$ containing a unit line segment in every direction has lower Minkowski dimension at least $(n+1)/2$ (Drury exponent)

Exercise 2 :

Let $d\sigma$ denote the rotation invariant measure on the sphere S in $\mathbf{R}^n$. Prove that

$$\int_S e^{-2\pi i \langle x,\xi \rangle}\, d\sigma = c_n \int_{-1}^{1} e^{-2\pi i|\xi|u}(1-u^2)^{(d-3)/2}\, du$$

for some constant c_n depending on the dimension n.

Exercise 3:

Consider the cylinder $C = \{(x_1,\dots,x_n) \in \mathbf{R}^n, \|(x_1,\dots,x_{n-1})\| = 1\}$ and let ϕ be a smooth function on $\mathbf{R}$ vanishing outside $(-1,1)$. Let $d\sigma$ be the hypersurface area on the cylinder. Define

$$\int_C \phi(x_n) e^{-2\pi i\langle x,\xi\rangle}\, d\sigma =: f(\xi)$$

For each unit vector $e \in \mathbf{R}^n$ discuss the asymptotics of $f(\lambda e)$ as $\lambda \to \infty$.