

1. Exercises, F RT WS 26/27

Tutorial Exercises (week 19.10. - 23.10.)

Exercise T1. The algebra

$$A := \left\{ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \mid a_{ij} \in K \right\}$$

of upper triangular matrices is isomorphic to the path algebra of the quiver

$$1 \xrightarrow{a} 2 \xrightarrow{b} 3$$

Construct this isomorphism. Try to realize the path algebra of

$$1 \xrightarrow{a} 2 \xleftarrow{b} 3$$

as an algebra of matrices.

Exercise T2. Consider the usual field extension $\mathbb{C}/\mathbb{R}$. This turns $\mathbb{C}$ into an $\mathbb{R}$ -algebra A . Show that A is not isomorphic to a path algebra $\mathbb{R}Q$.

Exercise T3. Let Q be a quiver. Show that the algebras $(KQ)^{\text{op}}$ and KQ^{op} are isomorphic.

Exercise T4. Let $A = K[T]/(T^m)$ for some $m \geq 2$. Use Linear Algebra to determine all finite-dimensional A -modules, up to isomorphism.

Homework Exercises (Deadline Fri 23.10., 15:00)

Exercise H1. Let

$$A := \begin{pmatrix} K[T]/(T^2) & K[T]/(T^2) \\ 0 & K \end{pmatrix}.$$

This is a K -algebra via the usual matrix multiplication. Show that A is isomorphic to KQ/I for some quiver Q and some ideal I of KQ .

Exercise H2. The **centre** $Z(R)$ of a ring R is defined as

$$Z(R) := \{c \in R \mid cr = rc \text{ for all } r \in R\}.$$

Let Q be a quiver. Determine the centre $Z(KQ)$.

Rules:

The maximal score for each homework exercise is 4 points. Students who get at least 50% of points and who participate actively in the tutorials are admitted to the exam. If a homework solution is submitted under your name, you have to be prepared to present the solution on the board. The tutors can choose who will present the solution.
