$G/\mathbb{Q}$ with $G(\mathbb{R})$ compact.

$$\mathcal{C} = C_0^\infty(G(\mathbb{Q})\backslash G(\mathbb{A}_f)) \circlearrowleft G(\mathbb{A}_f)$$

$$\mathrm{tr}(\mathbb{1}_{KgK};\mathcal{C}) = \mathrm{vol}(K)\cdot \#\,\mathrm{Fix}(g; G(\mathbb{Q})\backslash G(\mathbb{A}_f)/K)$$

$$\mathrm{Fix}_g = G(\mathbb{Q})\backslash\{(\bar{y},\gamma)\in (G(\mathbb{A}_f)/K\cap gKg^{-1})\times G(\mathbb{Q});\ ygK=\gamma gK\}$$

$$= \coprod_{\gamma\in G(\mathbb{Q})/\sim} G(\mathbb{Q})_\gamma\backslash\{y\in G(\mathbb{A}_f);\ y^{-1}gy\in KgK\}/K$$

$$\#\,\mathrm{Fix}_g = \sum_{\gamma\in G(\mathbb{Q})/\sim}\ \int_{G(\mathbb{A}_f)_\gamma\backslash G(\mathbb{A}_f)} \mathbb{1}_{KgK}(y^{-1}\gamma y)\,d\bar{y}$$

$$\mathrm{tr}(f;\mathcal{C}) = \sum_{\gamma\in G(\mathbb{Q})/\sim} O_\gamma(f)\quad ,\ f\in\mathcal{C}$$

$V = \mathbb{Q}_p^n$, basis $e_1, \dots, e_n$

$\Lambda_i = \mathrm{span}_{\mathbb{Z}_p} \{ p^{-1} e_1, \dots, p^{-1} e_i, e_{i+1}, \dots, e_n \}$

$\Lambda_0 \longrightarrow \Lambda_1 \longrightarrow \dots \longrightarrow \Lambda_{n-1} \xrightarrow{p} \Lambda_0$

$I = \{ i_0 < i_1 < \dots < i_{m-1} \} \subset \{ 0, \dots, n-1 \}$

$M_I(S)$ = set of commut. diagrams

$$\begin{array}{ccccccccc}
\Lambda_{i_0, S} & \longrightarrow & \Lambda_{i_1, S} & \longrightarrow & \dots & \longrightarrow & \Lambda_{i_{m-1}, S} & \longrightarrow & \Lambda_{i_0, S} \\
\cup & & \cup & & & & \cup & & \cup \\
\mathcal{F}_0 & \longrightarrow & \mathcal{F}_1 & \longrightarrow & \dots & \longrightarrow & \mathcal{F}_{m-1} & \longrightarrow & \mathcal{F}_0
\end{array}$$

$\mathcal{F}_i$ loc. direct summand, rank $\mathcal{F}_i = r$.

$$V = F^n \quad , \text{ basis } e_1, \dots, e_n$$

$$\phi : V \times V \longrightarrow F$$

$$\langle v, w \rangle = \operatorname{tr}_{F/\mathbb{Q}_p} \left(\varpi^{-1} \phi(v, w) \right)$$

$$\phi(e_i, e_{n+1-j}) = \delta_{ij}$$

$$\Lambda_i = \operatorname{span}_{\mathcal{O}_F} \{ \varpi^{-1} e_1, \dots, \varpi^{-1} e_i, e_{i+1}, \dots, e_n \}$$

$$\Lambda_{i+kn} = \varpi^{-k} \cdot \Lambda_i$$

$$\hat{\Lambda}_j = \Lambda_{-j}$$

$$I = \{ i_0 < i_1 < \dots < i_{m-1} \} \subset \{0, \dots, n-1\}$$

$$i \in I,\ i \neq 0 \Rightarrow n - i \in I.$$

$M_I^{\text{naive}}(S)$ = commutative diagrams

$$\begin{array}{ccccccccc}
\Lambda_{i_0,S} & \longrightarrow & \Lambda_{i_1,S} & \longrightarrow & \cdots & \longrightarrow & \Lambda_{i_{m-1},S} & \xrightarrow{\hat{\varpi}} & \Lambda_{i_0,S} \\
\cup & & \cup & & & & \cup & & \cup \\
\mathcal{F}_{i_0} & \longrightarrow & \mathcal{F}_{i_1} & \longrightarrow & \cdots & \longrightarrow & \mathcal{F}_{i_{m-1}} & \longrightarrow & \mathcal{F}_{i_0}
\end{array}$$

$\mathcal{F}_{i_j}$ loc. direct, $\mathcal{O}_F$-stable, $rk = n$

- isotropy condition
- $\mathrm{char}_{\hat{\varpi} | \mathcal{F}_{i_j}}(T) = (T - \hat{\varpi})^r \cdot (T - \overline{\hat{\varpi}})^s \in \mathcal{O}_E[T]$