Errata for [LRS]

We use the notations of [LRS].

In [LRS] the authors constructed for an automorphic representation Π of $D_{\mathbb{A}}^{\times}$ with $\Pi_\infty \simeq \mathrm{St}_d$ the Steinberg representation a $m(\Pi)d$-dimensional graded Galois representation $V^{\bullet}_{\Pi^\infty}$ as part of the cohomology of some moduli space for $\mathcal{D}$-elliptic sheaves. They claimed that the cup product makes $V^{\bullet}_{\Pi^\infty}$ selfdual. If Π^∞ is not selfdual up to some character twist, this is wrong. This false claim was only used in **(14.11) Corollary** and **(14.12) Theorem**.

(14.11) Corollary becomes correct, if we substitute $L_x(V^{\bullet}_{\Pi^\infty}, q_x^{-d}T^{-1})$ by $L_x(V^{\bullet\vee}_{\Pi^\infty}, q_x^{-1}T^{-1})$, in the statement as well as in the proof. Here we denote by $V^{\bullet\vee}_{\Pi^\infty}$ the dual Galois representation.

The proof of **(14.12) Theorem** (given in **(14.19)**, p.307) is correct word by word, if we substitute **(14.14) Lemma** and **(14.17) Proposition** by the following modifications (with the same proof for the Proposition$'$ and essentially the same proof for the Lemma$'$):

(14.14) Lemma$'$: Let us fix non-negative integers m and m' such that m' divides m. Let $V^\bullet$ be a pure graded Frob-semisimple l'adic representation of $\mathrm{Gal}(\overline{F}_\infty|F_\infty)$ (cf. **(14.13)**). We assume moreover, that

$$\frac{L_\infty(V^\bullet, T)}{L_\infty(V^{\bullet\vee}, q_\infty^{-1}T^{-1})} \sim \left(\frac{1-q_\infty^{-d}T^{-1}}{1-T}\right)^m,$$

and that

$$L_\infty(V^i, T)^{-1} \in (1 + T\overline{\mathbb{Q}}_l[T])^{m'}$$

for each $i = 0, \ldots, 2d-2$. Then there exists a direct summand $W^\bullet$ of $V^\bullet$ of the form

$$W^\bullet = \bigoplus_{\alpha \in A} W^\bullet_\alpha,$$

with

$$W^\bullet_\alpha \simeq \left[\bigoplus_{j=0}^{s} \sigma^0(\mathrm{St}_{i_j})(-i_0 - \ldots - i_{j-1})\right]^{m'}$$

($\mathrm{St}_i(-j)$ in degree $i+2j-1$) for some tuple $(i_0, \ldots, i_s)$ of positive integers with $i_0 + \ldots + i_s = d$, such that $m = |A|m'$.

(14.17) Proposition$'$: Assumption as in **(14.17) Proposition**, but the claim is:

$$V^\bullet = (V^{\bullet}_{\Pi^\infty})^{Frob-ss} \simeq \left[\bigoplus_{j=0}^{s} \sigma^0(\mathrm{St}_{i_j})(-i_0 - \ldots - i_{j-1})\right]^{m(\Pi)}$$

for some tuple $(i_0, \ldots, i_s)$ of positive integers with $i_0 + \ldots + i_s = d$.

Proof of **(14.14) Lemma$'$**: We make induction on m. The only irreducible Frob-semisimple representations U of $\mathrm{Gal}(\overline{F}_\infty|F_\infty)$ with $1-T$ dividing $L_\infty(U,T)^{-1}$ are of the form $\sigma^0(\mathrm{St}_i)$ for some i. Hence $\sigma^0(\mathrm{St}_{i_0})$ is a direct summand of $V^\bullet$ for some i_0. We remark that by purity the representation $\sigma^0(\mathrm{St}_i)(-j)$ can appear in $V^\bullet$ only in degree $i+2j-1$. Hence $\sigma^0(\mathrm{St}_{i_0})$ is actually a direct summand of V^{i_0-1}. Then $(1-T)^{m'}$ divides $L_\infty(V^{i_0-1}, T)^{-1}$, and by above, we see that $\sigma^0(\mathrm{St}_{i_0})^{m'}$ is a direct summand of V^{i_0-1}. Choosing some graded complement $V^{\bullet\prime}$ for this direct summand in $V^\bullet$, we get

$$\frac{L_\infty(V^{\bullet\prime}, T)}{L_\infty(V^{\bullet\prime\vee}, q_\infty^{-1}T^{-1})} \sim \left(\frac{1-q_\infty^{-d}T^{-1}}{1-T}\right)^{m-m'} \cdot \left(\frac{1-q_\infty^{-d}T^{-1}}{1-q_\infty^{-i_0}T^{-1}}\right)^{m'}.$$

If $i_0 = d$, we finish the proof by induction. Otherwise we conclude as above, that there is some positive integer i_1, such that $\sigma^0(\mathrm{St}_{i_1})(-i_0)^{m'}$ is a direct summand of $V^{i_1+2i_0-1\prime}$. Choosing again a complement $V^{\bullet\prime\prime}$, we get

$$\frac{L_\infty(V^{\bullet\prime\prime}, T)}{L_\infty(V^{\bullet\prime\prime\vee}, q_\infty^{-1}T^{-1})} \sim \left(\frac{1-q_\infty^{-d}T^{-1}}{1-T}\right)^{m-m'} \cdot \left(\frac{1-q_\infty^{-d}T^{-1}}{1-q_\infty^{-(i_0+i_1)}T^{-1}}\right)^{m'}.$$

Going on (and observing that this process has to stop), we get a direct summand W_α as in the claim. We finish the proof by induction.