

## ADVANCED MATHEMATICAL LOGIC I (V4A7) WISE 2026/27

### Tame geometry

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**Time and Place.** Mondays 2pm ct, Wednesdays 12pm ct We10/Kleiner Hörsaal

**E-Campus.** TBA

**Abstract.** This course is an introduction to tame geometry, with a particular focus on o-minimality. O-minimality was isolated by van den Dries [vdD84] to extend fundamental results from semi-algebraic geometry to a more general setting, and was developed by Pillay and Steinhorn [PS86] as a notion of tameness for dense linear orders. Among the many results that carry over from semi-algebraic geometry to o-minimal structures are the monotonicity theorem for definable functions and the cell decomposition theorem.

O-minimality has seen tremendous growth in its applications to number theory and algebraic geometry since Pila's proof of the unconditional case of the André–Oort conjecture for products of modular curves [Pil11]. More recently, o-minimality has also found substantial applications in Hodge theory (see, for example, Bakker, Klingler, and Tsimerman [BKT20], as well as Klingler's 2022 ICM talk [Kli21]). Although we will not have time to cover these applications in detail, you will learn all of the foundational material in o-minimality that underpins them. As a result, this course should also be of interest to students in algebra and geometry who wish to understand the role that o-minimality plays in these areas.

Throughout the course we will follow the excellent book by van den Dries [vdD98], covering most of Chapters 1–7. We will also discuss the Pila–Wilkie counting theorem [PW06], a landmark result in o-minimality that has driven many of its recent applications beyond mathematical logic.

**Prerequisites.** Only basic knowledge of first-order logic is assumed. Participation in the course *V3A5/F4A1 - Mathematical Logic* during the Wintersemester 2025/26 is surely helpful, but not required. The script for that course is available on the E-Campus website for this seminar.

**Literature.** Lecture notes will be provided. Excellent references are [vdD98] and [Bv22].

#### REFERENCES

- [BKT20] B. Bakker, B. Klingler, and J. Tsimerman, *Tame topology of arithmetic quotients and algebraicity of Hodge loci*, J. Amer. Math. Soc. **33** (2020), no. 4, 917–939. MR 4155216
- [Bv22] Neer Bhardwaj and Lou van den Dries, *On the pila–wilkie theorem*, Expositiones Mathematicae (2022).
- [Kli21] Bruno Klingler, *Hodge theory, between algebraicity and transcendence*, 2021.
- [Pil11] Jonathan Pila, *O-minimality and the André–Oort conjecture for  $\mathbb{C}^n$* , Ann. of Math. (2) **173** (2011), no. 3, 1779–1840. MR 2800724
- [PS86] Anand Pillay and Charles Steinhorn, *Definable sets in ordered structures. I*, Trans. Amer. Math. Soc. **295** (1986), no. 2, 565–592. MR 833697

- [PW06] J. Pila and A. J. Wilkie, *The rational points of a definable set*, Duke Math. J. **133** (2006), no. 3, 591–616. MR 2228464
- [vdD84] Lou van den Dries, *Remarks on Tarski's problem concerning  $(\mathbf{R}, +, \cdot, \text{exp})$* , Logic colloquium '82 (Florence, 1982), Stud. Logic Found. Math., vol. 112, North-Holland, Amsterdam, 1984, pp. 97–121. MR 762106
- [vdD98] ———, *Tame topology and o-minimal structures*, London Mathematical Society Lecture Note Series, vol. 248, Cambridge University Press, Cambridge, 1998. MR 1633348