GRADUATE SEMINAR ON LOGIC (S4A4) SOSE 2026

Topics in model theory

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Time and Place. Wednesday 10(c.t.)-12, SemR 0.006

Organizational Meeting. ~~Wednesday 4.2. 12(c.t.)~~ **Monday 2.2. 14 (c.t.)** Zeichensaal

If you want to give a talk, please send me (Philipp Hieronymi, `hieronymi@math.uni-bonn.de`) an email by Wednesday, 11.2.2026, indicating which talks you are interested in. If possible, list at least three topics.

Abstract. In this seminar we cover certain topics for which we did not have time during the V3A5 Mathematical Logic course, but which are very useful to know if you plan to specialize in model theory. The overall theme is to see more examples and to learn about some new notions from neostability theory. In general, we will focus more on particular examples than on the general theory.

Rules.

(1) In addition to giving a 90-minute talk, each participant (who wants to receive credit) is required to submit a summary of at least 4 pages of their topic, prepared using LaTeX, by the time they give their talk.
(2) Regular attendance is required.
(3) Seminar talks are graded. Your grade is determined (among other things) by the mathematical content, pedagogical effort, and quality of the presentation (for example, organized blackboard work and time management), as well as the written summary. You will often not be able to cover all the material in a single talk. It is your responsibility to choose the content of your talk wisely. (Which statements and definitions are important? How should they be illustrated via examples? Does it make sense to have a "running example"? Should you present a complete proof, only the idea of a proof, or no proof at all?)

Prerequisites. This seminar is designed for students who have taken *Mathematical Logic (V3A5/F4A1-5)* during the Winter Semester 2025/26. Thus, we assume knowledge of basic model theory. For talks (7) and (11a), some algebraic background knowledge is helpful.

Talks.

(1) **Differentially closed fields** (1 talk)
Cover the subsection *Quantifier elimination for differentially closed fields* in [7, p. 148]. Make sure to explicitly write down the axioms of DCF. Then cover [7, Exercise 4.5.43], including proofs. State the Ritt–Raudenbusch Basis Theorem, but do not prove it.

(2) **Stability and the order property** (1 talk)
Introduce the order property as in [7, Exercise 5.5.6]. State and prove [7, Lemma 5.2.12], and present the solution to [7, Exercise 5.5.6]. Then show that not having the order property implies stability. See also [10, Section 8.2].

(3) **NIP** (1 talk)
Cover the material of [9, Section 2.1] up to Example 2.12. Recall material from Chapter 1 where needed. Explain why o-minimal and stable theories are NIP.

(4) **NIP and VC-dimension** (1 talk)
Cover [9, Section 6.1].

(5) **Dp-rank** (1 talk)
Introduce mutually indiscernible sequences as in [9, Section 4.1]. Cover [9, Section 4.2]. Make sure to introduce ict-patterns [9, Definition 4.21] and state [9, Proposition 4.22]. See also the first few pages of [6].

(6) **Examples of structures with finite dp-rank** (1 talk)
Cover the appendix of [8], and then prove [2, 5.8]. Next, prove [1, Proposition 3.4], including the necessary lemmas and definitions. Finally, compute the dp-rank of the three theories.

(7) **Valuations** (1 talk)
Cover [3, Section 2.2]. Give proofs of the cited algebraic results if time permits. Also cover [3, Lemma 2.3.8].

(8) **The real field with a cyclic multiplicative subgroup** (1 talk)
Cover [3, Section 3.1], except for [3, Proposition 3.2.1].

(9) **Strong dependence** (1 talk)
Introduce the notion of strong dependence [9, Definition 4.23]. Explain [9, Example 4.24]. Introduce strongness as in [1, Section 1.1]. If time permits, show the equivalence between being strongly dependent as defined in [9] and being NIP and strong. Prove [1, Corollary 2.15].

(10) **Distality** (1 talk)
Cover [5, Section 1.1] (and indicate why o-minimal theories are distal) and [5, Section 2]. Then discuss [5, Section 3], including the cited material from [4, Section 6]. If time permits, explain why $(\mathbb{R}, <, +, \mathbb{Q})$ is not distal.

(11) **Quantifier elimination of $\mathbb{R}_{\mathrm{an}}$** (4–5 talks, if there are enough participants)
   (a) **Algebraic prerequisites** (2–3 talks)
   Cover Sections 3 through 8 of [11]. An algebraic background is needed for these talks.
   (b) **Quantifier elimination and o-minimality** (1–2 talks)
   Cover [11, Section 9].

References

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2. Alfred Dolich, Chris Miller, and Charles Steinhorn, *Structures having o-minimal open core*, Trans. Amer. Math. Soc. **362** (2010), no. 3, 1371–1411. MR 2563733
3. Leo Gitin, *Decidability in expansions of local fields*, `https://www.math.uni-bonn.de/people/phierony/Gitin.pdf`, 2022, Master thesis.
4. Ayhan Günaydin and Philipp Hieronymi, *Dependent pairs*, J. Symbolic Logic **76** (2011), no. 2, 377–390. MR 2830406
5. Philipp Hieronymi and Travis Nell, *Distal and non-distal pairs*, J. Symb. Log. **82** (2017), no. 1, 375–383. MR 3631293

6. Itay Kaplan, Alf Onshuus, and Alexander Usvyatsov, *Additivity of the dp-rank*, Trans. Amer. Math. Soc. **365** (2013), no. 11, 5783–5804. MR 3091265
7. David Marker, *Model theory*, Graduate Texts in Mathematics, vol. 217, Springer-Verlag, New York, 2002, An introduction. MR 1924282
8. Chris Miller, *Expansions of dense linear orders with the intermediate value property*, J. Symbolic Logic **66** (2001), no. 4, 1783–1790. MR 1877021
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10. Katrin Tent and Martin Ziegler, *A course in model theory*, Lecture Notes in Logic, vol. 40, Association for Symbolic Logic, La Jolla, CA; Cambridge University Press, Cambridge, 2012. MR 2908005
11. A. J. Wilkie, *Lectures on elimination theory for semialgebraic and subanalytic sets*, O-minimality and diophantine geometry, London Math. Soc. Lecture Note Ser., vol. 421, Cambridge Univ. Press, Cambridge, 2015, pp. 159–192. MR 3496445