Cardioids and Limacons*

Cardioids and Limacons are obtained if on the outside of one fixed circle of radius $r = aa$ another circle of the same radius rolls. These curves are traced by a radial stick of length $R = ii * r$, $ii = 1$ for Cardioids and $ii > 1$ for Limacons.

One choice of parametric equations for these curves is:

$$x(t) = 2r\cos(t) + R\cos(2t)$$
$$y(t) = 2r\sin(t) + R\sin(2t).$$

The evolute of the Cardioid is a smaller Cardioid, see in the Action Menu the entry `Show Osculating Circles with Normals`. In the entry `Add Caustics` one can rotate all normals by a fixed amount and these rotated lines always envelope a Cardioid.

To see the Cardioid generated by rolling a larger circle around a smaller one choose in the exhibit *Epi- and Hypocycloids* parameters $hh = 2 * aa$, $ii = 1$.

The image of the unit circle under the complex map

$$z \mapsto w(z) = z^2 + 2z$$

is a Cardioid; images of larger circles (around 0) are Limacons. Inverses $z \mapsto 1/w(z)$ of Limacons are figure-eight shaped, including a Lemniscate.