

Graduate Seminar (S4D3) Motivic Aspects of Hodge theory

Summer term 2023

Organized by Prof. Dr. D. Huybrechts

Start: April 11, Time: Tuesday 12:15-13:45. SR N 0.008.

Prerequisite for the seminar is a good knowledge of cohomology of complex manifolds, including basic facts on Chern classes and Hodge theory. (We do not assume or need the notion of Chow motives as developed in the winter term.) Depending on the audience, we will include more background material on (mixed) Hodge structures, constructible sheaves, etc.

Please contact me directly huybrech@ if you would like to participate. Registration is possible until all slots are taken.

The seminar will loosely follow the book [1] (which occasionally is a bit short on proofs). The talks will correspond roughly to the 14 Chapters and we will fill in necessary background material from other sources, e.g. [2,3,4,5,6,...]. (The following is a revision of the original program, in order to balance the various topics better.)

1. **Recollection from topology:** H^* , H_c^* , Mayer-Vietoris sequence and mapping cone (Section 2.1), classical Euler characteristic e vs. compact support Euler characteristic χ^c , Examples 1.1.1, Black-box Hodge decomposition, Hodge structures (Definition 1.2.1 & Examples 1.2.2), Cohomology of a blow-up (see Voisin's book).
Speaker: Michele Lorenzi
2. **Grothendieck ring of varieties:** Start with discussion in Section 1.1. Define $K_0(Var)$ (see Definition 2.1.3), Bittner's theorem (Theorem 2.1.5, Appendix A1 & A2).
Speaker: Mandy Kwok
3. **Applications of Bittner's theorem:** Define $K_0(\mathfrak{h}S)$ and prove Theorem 2.1.4. Appendix A3 (this needs some familiarity with Chow motives and would be presented as a black-box).
Speaker: Thomas Manopulo, **Coach:** Dominique Mattei
4. **Mixed Hodge structures:** Section 2.2 & Examples (Section 3).
Speaker: Samuel Lee
5. **Hodge theory revisited:** Reminder on cones, quasi-isomorphisms, higher direct images $Rf_*\mathbb{Q}$, de Rham complex, with Hodge filtration (Section 4 and relevant parts in Voisin's book).
Speaker: 9.5.2023. Jiexiang Huang
6. **Mixed Hodge theory revisited:** Section 5
Speaker: 16.5.2023. Mauro Varesco
7. **Motivic Hodge theory:**
Speaker: 6.6.2023, Aaron Wild
8. **Motivic aspects of degenerations:**
Speaker: 13.6. 2023, José Galindo Jimiénez, **Coach:** Evgeny Shinder
9. **Motivic nearby fibres:** Section 8
Speaker: 27.6.2023, Jonas Walter, **Coach:** Evgeny Shinder

10. **Applications to singularities:** Section 9
Speaker: 4.7.2023, Jan Fornal, **Coach:** Yagna Dutta
11. **Motives in the relative setting:**
Speaker: 11.7.2023, Moritz Hartlieb
12. **Variations of Hodge structures:**
Speaker: 18.7.2023, Alessandro Nanto, **Coach:** Yoonjoo Kim

References:

1. Chris Peters *Motivic aspects of Hodge theory*. Narosa Publishing House. 2010 A draft of the book can be found here <https://www-fourier.ujf-grenoble.fr/~peters/Books/motivic.f/Tatalects.pdf>
2. A. Durfee *A naive guide to mixed Hodge structures*, Proc. Symp. in Pure Math. Vol. 40/I (1983) 313-320.
3. Robert Friedman *Mixed Hodge structures*. Notes <http://www.math.columbia.edu/~rf/MHS.pdf>.
4. Sergey Gelfand, Yuri Manin *Homological algebra*. Springer Verlag 1999, Chapter 6.
5. Daniel Huybrechts, *Complex Geometry*, Springer, Berlin, 2005.
6. Chris Peters & Joseph Steenbrink *Mixed Hodge Structures*. Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge 52. 2008.
7. Claire Voisin *Hodge Theory and complex algebraic geometry I, & II*. Cambridge University Press, Cambridge, 2002.