# Problem sheet 2 for Rigid Analytic Geometry Winter Term 2026/27

We present two applications of the Baire principle from the previous exercise sheet.

Let $A$ be a complete nat-ring with a countable neighbourhood base of 0. For instance, $A$ might be a Huber ring. For $s \in A$, an ideal $I \subseteq A$ is called $s$-saturated if the inclusion

$$I \subseteq I : s = \{a \in A \mid sa \in I\}$$

is an equality.

**Problem 1** (6 points)**.** *Assume that $s \in A^{\circ\circ}$ and that every $s$-saturated ideal in $A$ is closed in $A$. Then every sequence*

$$I_0 \subseteq I_1 \subseteq$$

*of $s$-saturated ideals in $A$ terminates in some $j \in \mathbb{N}$ with $I_j = I_k$ for $k \geq j$.*

**Definition 1.** *We call a nat-Ring $A$* rigid Noetherian *if $A_s$ is Noetherian for all $s \in A^{\circ\circ}$.*

**Remark 1.** *For Huber rings this coincides with the notion introduced in B. Zavyalov, Sheafiness of strongly rigid-Noetherian Huber pairs.*

**Problem 2** (1 point)**.** *Assume that $A$ is analytic in the sense that the ideal of $A$ generated by $A^{\circ\circ}$ equals $A$. Show that $A$ is rigid Noetherian if and only if it is Noetherian.*

In particular, the two notions coincide in the case of Tate rings.

**Problem 3** (2 points)**.** *Assume that $s \in A^{\circ\circ}$ and that all ideals of $A$ are closed in $A$. Show that $A$ is rigid Noetherian!*

In particular, if all ideals of a complete Tate ring $A$ are closed then $A$ is Noetherian, as was claimed in the lecture.

**Problem 4** (4 points)**.** *Let $A$ be a nat-ring with a topologically nilpotent unit and $M \xrightarrow{f} N$ a surjective morphism of nat-$A$-modules where $N$ has a countable neigbourhood base of 0 and is complete. For every open subgroup $U \subseteq M$, show that the closure $\overline{f(U)}$ of its image in $N$ is an open subgroup of $N$!*

The following is a form of the Banach Open Mapping Theorem suitable for our purposes.

**Problem 5** (7 points)**.** *In the situation of the previous problem, assume moreover that $M$ also has a countable neigbourhood base of 0 and is complete. Show that $f$ is an open map!*

Solutions should be e-mailed to your tutor s45qhou at uni hyphen bonn dot de before Friday October 30. Good luck!