

PROBLEM SHEET 1 FOR RIGID ANALYTIC GEOMETRY WINTER
TERM 2026/27

Let A be a sequentially complete nat-ring.

Problem 1 (4 points). *Show that the principal ideal of $A\langle X, Y \rangle$ generated by $XY - 1$ is closed.*

Under the same assumptions on A , let $A\langle T, T^{-1} \rangle$ be the ring of all formal Laurent series

$$(1) \quad \sum_{k=-\infty}^{\infty} f_k T^k$$

with

$$(2) \quad \lim_{k \rightarrow \pm\infty} f_k = 0$$

and where the product is given by

$$(3) \quad (fg)_k = \sum_{l=-\infty}^{\infty} f_l g_{k-l}$$

which is easily seen to converge. For an open subgroup U of A^1 , let $U\langle T, T^{-1} \rangle$ be the set of all formal Laurent series (1) such that all $f_k \in U$ and (2) holds. We use the set of these subgroups as a neighbourhood base of 0.

Problem 2 (5 points). *Show that $A\langle T, T^{-1} \rangle$ is a sequentially complete nat A -algebra!*

Mostly this is trivial but completeness, the continuity of (3) and associativity should be verified with reasonable care.

Problem 3 (4 points). *Show that there is a unique continuous morphism*

$$A\langle X, Y \rangle \rightarrow A\langle T, T^{-1} \rangle$$

of A -algebras sending X to T and Y to T^{-1} , and that it defines an isomorphism $A\langle X, Y \rangle / (XY - 1) A\langle X, Y \rangle \xrightarrow{\cong} A\langle T, T^{-1} \rangle$.

In Proposition 4.2.1 of the lecture only a closedness assumption for principal ideals was needed. In the case of complete Tate rings (in the sense of Huber, i. e., one has an open bounded subring and a topologically nilpotent unit), this is equivalent to Noetherianness. For this, the following version of Baire's principle is needed:

¹More precisely, of $(A, +)$

Problem 4 (7 points). *Let X be a complete abelian top-group with a countable neighbourhood base of 0, and let $(A_i)_{i=1}^{\infty}$ be a family of closed subgroups of X such that $\bigcup_{i=1}^{\infty} A_i$ is an open subgroup of X . Then at least one of the subgroups A_i is open in X .*

Solutions should be e-mailed to your tutor s45qhou at uni hyphen bonn dot de before Friday October 23. Good luck for this last exercise sheet, and good luck for the exam!