Problem 1 (6 points)**.** *Let X be a spectral space. Show that X is Noetherian if and only if for every spectral space Y, every continuous map $X \xrightarrow{f} Y$ is spectral!*

Problem 2 (2 points)**.** *Let $\mathfrak{P}$ be a prime cone of the ring R and $\mathfrak{p} = \operatorname{supp}\mathfrak{P}$. Show that every $\mathfrak{P}$-convex ideal of R contains $\mathfrak{p}$.*

Problem 3 (12 points)**.** *Let $\mathfrak{P}$ and $\mathfrak{p}$ be as in the previous exercise. Show that the following defines a bijection between the set of prime cones $\mathfrak{Q}$ of R with $\mathfrak{Q} \supseteq \mathfrak{P}$ and the set of $\mathfrak{P}$-convex prime ideals $\mathfrak{q}$ of R:*

- *A prime cone $\mathfrak{Q}$ containing $\mathfrak{P}$ is sent to $\mathfrak{q} = \operatorname{supp}\mathfrak{Q}$.*
- *A $\mathfrak{P}$-convex prime ideal $\mathfrak{q}$ is sent to $\mathfrak{Q} = \mathfrak{q} \cup \mathfrak{P}$.*

Solutions should be submitted in the lecture Friday, April 19.