Graduate Seminar on Algebraic Geometry
S4A1: Chow groups and motives

Organizers: D. Huybrechts & D. Mattei
Time: to be fixed
To register for the class please write an email to dmattei@ (Bonn uni adress) explaining your background in algebraic geometry and with a list of preference for a talk (3 talks ranked).
Deadline for registration: 31 August. Distribution of talks shortly afterwards. Later registration will be considered if slots are still available.
Preparation of talks: Talks will be 90 min but plan time for questions. Two weeks before delivering the talk you have to present a detailed plan for it to D. Mattei who will also answer all remaining questions.

Plan for the seminar

Main references: [EH16] for basic material on Chow groups, with many examples to play with, [Voi07] for more advanced topics in Chow groups (a french version of the book also exists), and [MNP13] for the theory of Chow motives. Two examples will be studied in independant articles ([BV04] and [Lat17]).

Chow groups The goal of this part is to define the basic material of Chow groups, give plenty of examples and state some applications of the theory: link between 0-cycles and Albanese variety, dimension of Chow groups and Hodge numbers. Finally, we study the Mumford theorem, stating that under cohomological condition, the 0-th Chow group of a surface is infinite dimensional.

1. Basics on Chow groups ([EH16] sections 1.2.1, 1.2.2, 1.2.3, we admit the moving lemma for this talk). Proper pushforward and flat pullback ([EH16] section 1.3.6 or [Voi07] section 9.1.2). First examples of computations: affine and projective spaces (and their product) ([EH16] sections 1.3.3, 2.1, 2.1.4). First applications: Bézout, degree of Veronese, degree of dual hypersurface (sections 2.1.1-3).
2. Intersection in Chow groups ([EH16], sections 1.2.3, 1.2.4). The Moving Lemma ([EH16], appendix A.1-2). Projection formula, cycle class map and compatibilities ([Voi07] sections 9.2.3 and 9.2.4).
3. Chow group of projective bundle ([EH16], Theorem 9.6), Chern classes of vector bundles ([EH16], Theorem 5.3, Definition 5.10). Chow group of a blow-up ([EH16], section 13.6.2, example section 13.6.3).
4. Dimension/representability of $\mathrm{CH}_0(X)$ ([Voi07] section 10.1.1), Roitman theorem ([Voi07] section 10.1.2).
5. Decomposition of the diagonal (Bloch-Srinivas) ([Voi07], section 10.2.1), and Mumford infinite dimension theorem (Theorem 10.15, proof)

6. The Beauville-Voisin class in CH_0 of a K3 surface, following [BV04].

Chow motives In this second part, we introduce the notion of Chow motives, an object standing between a variety and its cohomology. We exhibit explicit examples. Then we state different conjectures, with examples for which they hold, and we state some of their consequences. All references (when not stated otherwise) are from [MNP13]. We can restrict ourselves to $\mathbb{C}$.

7. Definition of adequate equivalence relation (section 1.2: rational (already seen), algebraic, homological and Weil cohomology (Betti and De Rham), numerical). Sections 2.1 to 2.5: definition of correspondances and motives (we could focus on rational equivalence), first examples and properties, direct sum and tensor product, Chow groups and cohomology.

8. More examples: motives of curves and Jacobians (section 2.7). Manin's identity principle and motives of projective bundles and blow-ups (section 2.8).

9. Dimension of motives (section 4.4, we might recall the needed material from section 4.1-3 depending on what we need). Finite dimension of curves (section 4.6). Section 5.4: surjective morphism and finite dimensionality, consequences of finite dimensionality of abelian varieties (corollary 5.4.7). Kimura-O'Sullivan conjecture of finite dimensionality (conjecture 5.6.9).

10. (Chow)-Künneth decomposition (section 6.1.1, 6.1.2), Picard and Albanese motives (theorem 6.2.1), treatment of the case of surfaces (section 6.3.2), application to Bloch conjecture (Proposition 5.6.13).

11. Bloch-Beilinson conjectural filtration (section 7.1), and the conjectures I-IV (section 7.2, we admit the equivalence theorem 7.5.1). Some general consequences (proposition 7.5.2, corollaries 7.5.3-10). Example of product of surfaces: proposition 7.6.1 and corollary 7.6.2.

12. Motive of the variety of lines in a cubic hypersurface, following [Lat17].

References

[BV04] Arnaud Beauville and Claire Voisin. On the Chow ring of a $K3$ surface. *J. Algebr. Geom.*, 13(3):417–426, 2004.

[EH16] David Eisenbud and Joe Harris. *3264 and all that—a second course in algebraic geometry*. Cambridge University Press, Cambridge, 2016.

[Lat17] Robert Laterveer. A remark on the motive of the Fano variety of lines of a cubic. *Ann. Math. Qué.*, 41(1):141–154, 2017.

[MNP13] Jacob P. Murre, Jan Nagel, and Chris A. M. Peters. *Lectures on the theory of pure motives*, volume 61 of *University Lecture Series*. American Mathematical Society, Providence, RI, 2013.

[Voi07] Claire Voisin. *Hodge theory and complex algebraic geometry. II*, volume 77 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, english edition, 2007. Translated from the French by Leila Schneps.