Exercises for Algebraic Topology I – Sheet 9

Uni Bonn, WS 2018/19

Aufgabe 33. Decide which of the following spaces are Eilenberg-Mac-Lane spaces. If the answer is yes, determine for which G and n they are of the type $K(G, n)$.

(a) A connected orientable closed surface of genus ≥ 1;

(b) $\mathbb{CP}^\infty$;

(c) The total space E of a fibration $p\colon E \to S^1$ with fiber $\mathbb{RP}^\infty$;

(d) $S^3 \times S^2$;

(e) A 4-dimensional connected closed manifold with infinite cyclic fundamental group.

Aufgabe 34. Let $p\colon S^{2n+1} \to \mathbb{CP}^n$ be the map sending $z \in S^{2n+1} \subseteq \mathbb{C}^{n+1}$ to the 1-dimensional complex vector space of $\mathbb{C}^{n+1}$ generated by z. Prove or disprove:

(a) p is a principal S^1-bundle;

(b) p admits a section, i.e., a map $s\colon \mathbb{CP}^n \to S^{2n+1}$ with $p \circ s = \mathrm{id}_{\mathbb{CP}^n}$.

Aufgabe 35. Let M be a connected closed oriented 3-manifold. Which of the following assertions are equivalent?

(a) M is homotopy equivalent to S^3;

(b) M is simply connected;

(c) There is a map $S^3 \to M$ of degree one.

Aufgabe 36. Let M be a closed connected oriented n-manifold. Let G be a non-trivial finite group which acts freely and orientation preserving on M. Equip S^n with the trivial G-action.

Prove or disprove that the forgetful map

$$[M, S^n]^G \to [M, S^n]$$

is never surjective, where $[M, S^n]^G$ denotes the G-homotopy classes of G-equivariant maps from M to S^n.

handover on Wednesday, 11.12 in the lecture