Exercises for Algebraic Topology I – Sheet 7

Uni Bonn, WS 2018/19

Aufgabe 25. Let G be a path connected topological group. Show that G is simple.

Aufgabe 26. Let G be a path connected topological group and $p\colon E \to B$ be a principal G-bundle. Prove or disprove:

(a) Its fiber transport is equivalent to the constant functor from $\Pi(B)$ to the homotopy category of spaces sending every object in $\Pi(B)$ to G;

(b) The group $H_k^{\Pi}(B; \pi_n(\mathcal{G})) = H_k(C_*(\widetilde{B}) \otimes_{\mathbb{Z}\Pi(B)} \pi_n(\mathcal{G}))$ is isomorphic to $H_k(B; \pi_n(G)) = H_k(C_*(B) \otimes_{\mathbb{Z}} \pi_n(G))$ for every $k \geq 0$ and $n \geq 1$, where $\pi_n(\mathcal{G})$ is the local coefficient system coming from the n-homotopy groups of the fibers.

Aufgabe 27. Let M be a closed connected n-dimensional manifold. Let o_{TM} be the $\mathbb{Z}\Pi(M)$-module associated to its tangent bundle. Compute $H_n^{\Pi}(M; o_{TM})$.

Aufgabe 28. Compute the homotopy group $\pi_2(X)$ as a module over $\mathbb{Z}[\pi_1(X)]$ for $X = S^1 \vee \mathbb{CP}^{\infty}$.

Please note: The student council of mathematics will organize the Maths Party on 28/11 in N8schicht. The presale will be held on Tue 26/11, Wed 27/11 and Thu 28/11 in the mensa Poppelsdorf. Further information can be found on fsmath.uni-bonn.de.

handover on Wednesday, 27.11 in the lecture