Exercises for Algebraic Topology I – Sheet 6

Uni Bonn, WS 2018/19

Aufgabe 21. Let $f, g\colon X \to Y$ be two homotopic maps.

(a) Show that the functors $\Pi(f)$ and $\Pi(g)$ from $\Pi(X)$ to $\Pi(Y)$ are naturally equivalent.

(b) Suppose that X and Y are connected and suppose that there are points $x \in X$ and $y \in Y$ with $y = f(x) = g(x)$. Prove or disprove that the functors $\Pi(f)$ and $\Pi(g)$ from $\Pi(X)$ to $\Pi(Y)$ are naturally equivalent if and only if the two group homomorphism $\pi_1(f, x)$ and $\pi_1(g, x)$ from $\pi_1(X, x)$ to $\pi_1(Y, y)$ differ by an inner automorphism of $\pi_1(Y, y)$.

Aufgabe 22. Let G be a path connected topological group.

(a) Construct an isomorphism of abelian groups $\pi_n(BG) \xrightarrow{\cong} \pi_{n-1}(G)$ for $n \geq 1$ and show that BG is simply connected.

(b) Suppose that G is $(k-1)$-connected. Prove or disprove that any principal G-bundle $p\colon E \to B$ over a k-dimensional CW-complex B is trivial.

Aufgabe 23. (a) Let $f\colon E \to B$ be an injective map with a path connected space as target and a non-empty space as source. Suppose that f is a fibration.

Does this imply that f is a weak homotopy equivalence?

(b) Let $f\colon E \to B$ be a surjective map which is a cofibration. Does this imply that f is a homeomorphism?

Aufgabe 24. Classify all principal S^1-bundles over S^n for $n \geq 1$.

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handover on Wednesday, 20.11 in the lecture