Exercises for Algebraic Topology I – Sheet 5

Uni Bonn, WS 2018/19

Aufgabe 17. Let M be a covariant $R\mathcal{C}$-module. Let $\underline{R}$ be the constant contravariant $R\mathcal{C}$-module with value R. Show that $\operatorname{Tor}_0^{R\mathcal{C}}(\underline{R}; M)$ is R-isomorphic to the colimit $\operatorname{colim}_{\mathcal{C}} M$.

Aufgabe 18. Let $F, G\colon \mathcal{C} \to \mathcal{D}$ be functors. Suppose that $T\colon F \xrightarrow{\cong} G$ is a natural equivalence. Show that it induces natural equivalences $T_*\colon F_* \xrightarrow{\cong} G_*$ and $T_!\colon F_! \xrightarrow{\cong} G_!$ of functors $R\mathcal{C} - \mathrm{MOD} \to R\mathcal{D} - \mathrm{MOD}$ and a natural equivalence $T^*\colon F^* \xrightarrow{\cong} G^*$ of functors $R\mathcal{D} - \mathrm{MOD} \to R\mathcal{C} - \mathrm{MOD}$

Aufgabe 19. Let M and N be two closed manifolds of dimension ≤ 2. Prove or disprove that M and N are homeomorphic if and only if their fundamental groupoids $\Pi(M)$ and $\Pi(N)$ are equivalent.

Aufgabe 20. Which of the following statements are true for all functors $F\colon \mathcal{C} \to \mathcal{D}$? (Give a reason for your answers.)

(a) F_* is exact;

(b) We obtain a homomorphism $F_*\colon K_0(R\mathcal{C}) \to K_0(R\mathcal{D})$ by $[P] \mapsto [F_*P]$;

(c) Suppose that F is surjective on objects and on morphisms. Then we obtain a well-defined homomorphism $F^*\colon G_0(R\mathcal{D}) \to G_0(R\mathcal{C})$ by $[M] \mapsto [F^*M]$;

(d) $F_!$ is right exact.

handover on Wednesday, 13.11 in the lecture