Exercises for Algebraic Topology I – Sheet 4

Uni Bonn, WS 2018/19

Aufgabe 13. Let N be a covariant $R\mathcal{C}$-module. Show that the functors

$$\begin{aligned}
\mathrm{MOD}\text{-}R\mathcal{C} &\to R\text{-}\mathrm{MOD}, & M &\mapsto M \otimes_{R\mathcal{C}} N, \\
R\text{-}\mathrm{MOD} &\to \mathrm{MOD}\text{-}R\mathcal{C}, & L &\mapsto \hom_R(N, L),
\end{aligned}$$

form an adjoint pair.

Aufgabe 14. Let $\mathcal{C}$ be a finite category, i.e., it has only finitely many objects and morphisms. Let R be a Noetherian ring.

Prove or disprove that the category $R\mathcal{C}$ is Noetherian, i.e., any $R\mathcal{C}$-submodule of a finitely generated $R\mathcal{C}$-module is finitely generated again.

Here, an $R\mathcal{C}$-module M is called finitely generated if there is a finite set $S \subseteq \bigcup_{c \in \mathcal{C}} M(c)$ such that the smallest $\mathbb{R}\mathcal{C}$-submodule of M containing S is M itself.

Aufgabe 15. Let I be the category having two objects 0 and 1 and three morphisms $\mathrm{id}\colon 0 \to 0$, $\mathrm{id}\colon 1 \to 1$ and $u\colon 0 \to 1$. Consider the statements:

(a) M is a projective RI-module;

(b) $M(u)\colon M(0) \to M(1)$ is injective, and both $M(0)$ and the cokernel of $M(u)$ are projective R-modules.

Show (a) $\implies$ (b). Prove or disprove the implication (b) $\implies$ (a).

Aufgabe 16. Show that a 1-dimensional vector bundle ξ is trivial if and only if its first Stiefel-Whitney class $w_1(\xi)$ is trivial.

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handover on Wednesday, 06.11 in the lecture