Exercises for Algebraic Topology I – Sheet 13

Uni Bonn, WS 2018/19

Aufgabe 49. Consider a 1-form ω on $S^1 \times S^1$. Show that ω is exact if and only if it is closed and $\int_{S^1} i_k^*\omega$ vanishes for $k = 1, 2$, where $i_k \colon S^1 \to S^1 \times S^1$ is given by $i_1(z) = (z, z_0)$ and $i_2(z) = (z_0, z)$ for some fixed base point $z_0 \in S^1$.

Aufgabe 50. Let M and N be oriented closed connected smooth manifolds of dimension n and let $f \colon M \to N$ be a smooth map. Prove or disprove that for any smooth map $f \colon M \to N$ and n-form ω on N we have

$$\int_M f^*\omega = \mathrm{degree}(f) \cdot \int_N \omega.$$

Aufgabe 51. Let $\omega \in \Omega^2\mathbb{CP}^n$ be a 2-form which is closed but not exact. Prove or disprove that the k-fold product $\omega \wedge \omega \wedge \cdots \wedge \omega$ is exact if and only if $k > n$ holds

Aufgabe 52. Let M be a closed connected orientable manifold of dimension n. Decide which of the following assertions are equivalent:

(a) For every natural number m with $1 \leq m \leq n-1$ every closed m-form is exact;

(b) For every natural number m with $1 \leq m \leq n-1$ the abelian group $H_m(M;\mathbb{Z})$ is finite;

(c) For every natural number m with $1 \leq m \leq n-1$ the abelian group $H^m(M;\mathbb{Z})$ is finite.

handover on Wednesday, 21.01 in the lecture