

Exercises for Algebraic Topology I – Sheet 11

Uni Bonn, WS 2018/19

Aufgabe 41. Decide for which $m, n \geq 1$ every map $\mathbb{R}P^m \rightarrow K(\mathbb{Z}/3, n)$ is homotopic to the constant map.

Aufgabe 42. Consider an abelian group G and $n \geq 1$. Let Y be a space such that for every CW -complex X there exists an isomorphism, natural in X , $[X, Y] \xrightarrow{\cong} H^n(X; G)$. Prove or disprove that Y is an Eilenberg-MacLane space of type (G, n) .

Aufgabe 43. Classify all principal S^1 -bundles over $\mathbb{R}P^3$ up to isomorphism and give for every isomorphism class an explicit representative.

Aufgabe 44. Let F be a CW -complex and assume that F is homotopy equivalent to a loop space. Decide which of the following assertions are equivalent:

- (a) F is contractible.
- (b) Every fibration $p: E \rightarrow B$ over a CW -complex B with typical fiber F has a section.
- (c) Every fibration $p: E \rightarrow B$ over a CW -complex B with typical fiber F is up to fiber homotopy trivial.