Exercises for Topology I – Sheet 9

University of Bonn, WS 2018/19

Exercise 33. Let (X, A) be a relative CW-complex. Let $\mathcal{H}_*$ be a homology theory with values in R-modules. Show that $(X/A, A/A)$ is a CW-pair and that the projection $(X, A) \to (X/A, A/A)$ induces for all n an R-isomorphism $\mathcal{H}_n(X, A) \xrightarrow{\cong} \mathcal{H}_n(X/A, A/A)$.

Exercise 34. Let X and Y be finite CW-complexes. Show that the CW-complex $X \times Y$ is finite and prove

$$\chi(X \times Y) = \chi(X) \cdot \chi(Y).$$

Exercise 35. Compute the Euler characteristic of the finite CW-complex X, where X is given by S^n, $\mathbb{RP}^n$, or $\mathbb{CP}^n$ for $n \geq 0$, the orientable surface F_g of genus $g \geq 0$, or X is a finite, connected, 2-dimensional CW-complex whose fundamental group is the symmetric group S_5 and $H_2(X; \mathbb{F}_5)$ has cardinality 5 for $\mathbb{F}_5$ the field with 5 elements.

Exercise 36. Let Δ_n be the standard n-simplex. Compute $\chi(\Delta_n)$. Show that the result implies the formula

$$\sum_{k=0}^{n} (-1)^k \cdot \binom{n}{k} = 0.$$

to be handed in on 10.12. during the lecture