Exercises for Topology I – Sheet 7

University of Bonn, WS 2018/19

Exercise 25. Let X be a finite CW-complex. Let $\mathcal{H}_*$ be a homology theory with values in $\mathbb{Z}$-modules which satisfies the dimension axiom and for which $\mathcal{H}_0(\bullet)$ is a finitely generated abelian group. Show that $\mathcal{H}_n(X)$ is finitely generated for all $n \in \mathbb{Z}$ and that there is a natural number d with $\mathcal{H}_n(X) = 0$ for $n \geq d$ and for $n \leq -1$.

Exercise 26. Compute the singular homology of $\mathbb{CP}^m \times \mathbb{CP}^n$ with coefficients in the ring R for any $m, n \geq 0$.

Exercise 27. Let X be a 2-dimensional finite CW-complex whose fundamental group and whose second singular homology group with values in $\mathbb{Z}$-modules are finite, and which has precisely one 0-cell.

Show that the number of 1-cells and the number of 2-cells agree.

Exercise 28. Show that any CW-structure on $S^1 \times S^1$ has at least four cells.

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to be handed in on 26.11. during the lecture