# Exercises for Topology I – Sheet 6

## University of Bonn, WS 2018/19

**Exercise 21.** Let $\{F_n \mid n \in \mathbb{Z}\}$ be a collection of free $R$-modules. Prove or disprove that there is a homology theory $\mathcal{H}_*$ satisfying the disjoint union axiom such that $\mathcal{H}_n(\{\bullet\}) \cong F_n$ holds for $n \in \mathbb{Z}$.

**Exercise 22.** Show that the Hawaiian earring is not a $CW$-complex. (If you do not know what the Hawaiian earring is, check the literature.)

**Exercise 23.** Show for a $CW$-complex $X$ that the following statements are equivalent:

(a) For every $n \geq 1$, every map $S^{n-1} \to X$ can be extended to a map $D^n \to X$.

(b) Every map $Y \to X$ from a $CW$-complex $Y$ to $X$ is homotopic to a constant map.

(c) $X$ is contractible.

(d) Every map $Z \to X$ from a topological space $Z$ to $X$ is homotopic to a constant map.

**Exercise 24.** Compute the singular homology of $\mathbb{CP}^d$ for $d \geq 1$.

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to be handed in on 19.11. during the lecture