Exercises for Topology I – Sheet 5

University of Bonn, WS 2018/19

Exercise 17. Decide whether there is a path connected topological space whose fundamental group is isomorphic to the symmetric group S_5 of permutations of the set of five elements and whose first singular homology vanishes.

Decide the same question after replacing S_5 by the subgroup A_5 of even permutations.

Exercise 18. Let $\mathcal{H}_*$ be a homology theory with values in $\mathbb{Z}$-modules satisfying the disjoint union axiom and the dimension axiom. Consider a sequence $A_1, A_2, \ldots$ of finitely generated abelian groups.

Construct a path connected space X satisfying $\mathcal{H}_i(X) \cong_{\mathbb{Z}} A_i$ for $i = 1, 2, \ldots$.

Exercise 19. Let F be a field and let C_* be a finite positive F-chain complex of dimension $\leq d$. Prove

$$\sum_{n=0}^{d} (-1)^n \cdot \dim_F(C_n) = \sum_{n=0}^{d} (-1)^n \cdot \dim_F(H_n(C_*)).$$

Exercise 20. Let d be any natural number satisfying $d \geq 1$. Let $\mathcal{H}_*$ be a homology theory with values in $\mathbb{Z}$-modules satisfying the dimension axiom. Consider any selfhomomorphism $u\colon \mathcal{H}_d(S^d) \to \mathcal{H}_d(S^d)$.

Prove or disprove that there is a selfmap $f\colon S^d \to S^d$ satisfying $\mathcal{H}_d(f) = u$.

to be handed in on 12.11. during the lecture