Exercises for Topology I – Sheet 3

University of Bonn, WS 2018/19

Exercise 9. (a) Let C_* be a projective positive R-chain complex. Show that C_* is acyclic if and only if it is contractible.

(b) Give an example of a finite 2-dimensional positive $\mathbb{Z}$-chain complex which is acyclic but not contractible.

Exercise 10. Consider a commutative diagram of R-modules with exact rows

$$\begin{tikzcd} 0 \arrow[r] & M_0 \arrow[r] \arrow[d, "f_0"] & M_1 \arrow[r] \arrow[d, "f_1"] & M_2 \arrow[r] \arrow[d, "f_2"] & 0 \\ 0 \arrow[r] & N_0 \arrow[r] & N_1 \arrow[r] & N_2 \arrow[r] & 0. \end{tikzcd}$$

Show the existence of a long exact sequence

$$0 \to \ker(f_0) \to \ker(f_1) \to \ker(f_2) \to \operatorname{coker}(f_0) \to \operatorname{coker}(f_1) \to \operatorname{coker}(f_2) \to 0.$$

Exercise 11. (a) Let M be d-dimensional topological manifold for $d \geq 1$. Let $\mathcal{H}_*$ be a homology theory with values in R-modules satisfying the dimension axiom. Show that $\mathcal{H}_d(M, M - \{x\})$ is R-isomorphic to R for every point $x \in M$.

(b) Prove that $S^d \vee S^d$ is not a topological manifold for $d \geq 1$

Exercise 12. Let $\mathcal{H}_*$ be a homology theory with values in $\mathbb{Z}$-modules satisfying the dimension axiom. Consider a finitely generated abelian group A.

Construct a path connected finite 2-dimensional CW-complex X with $\mathcal{H}_1(X) \cong A$.

to be handed in on 29.10. during the lecture