FRIABLE VALUES OF BINARY FORMS

ANTAL BALOG, VALENTIN BLOMER, CÉCILE DARTYGE, AND GÉRALD TENENBAUM

Abstract. Let $F \in \mathbb{Z}[X,Y]$ be an integral binary form of degree $g \geq 2$, and let

$$\Psi_F(x,y) := \text{card}\{1 \leq a, b \leq x : P^+(F(a,b)) \leq y\}$$

where as usual $P^+(n)$ denotes the largest prime factor of $n$. It is proved that $\Psi_F(x,y) \asymp x^2$ for $y = x^{g-2+\epsilon}$ in general, and $y = x^{1/\sqrt{e}+\epsilon}$ if $g = 3$. Better results are obtained if $F$ is reducible.

To the memory of our friend and colleague George Greaves

2000 Mathematics Subject Classification. 11E76, 11N25, 11N36, 11Y05.

Key words and phrases. Friable integers, binary forms, sieves.