Graduate Seminar on Advanced Algebra

Supermathematics

Summer term 2020

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The word *super* refers in mathematics and physics to $\mathbf{Z}_2$-graded structures: Supergroups, Superalgebras, Supermanifolds, Superwhatever. The seminar will focus on the structure and representation theory of Lie superalgebras and supergroups.

A Lie superalgebra is a $\mathbf{Z}_2$-graded vector space $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ with a super bracket operation akin to the one of a Lie algebra, but with extra sign factors thrown in according to the parity of the elements. The classical Lie algebras of type $ABCD$ have super counterparts: The Lie algebra $\mathfrak{sl}(n)$ gets replaced by the Lie superalgebra $\mathfrak{sl}(m|n)$ and the Lie algebras $\mathfrak{o}(m)$ and $\mathfrak{sp}(2n)$ fuse into the *orthosymplectic* Lie superalgebra $\mathfrak{osp}(m|2n)$.

Initially, the story seems to be parallel to the classical theory: The interplay between Lie superalgebras and algebraic supergroups is like for ordinary Lie algebras, simple Lie superalgebras over $\mathbb{C}$ are classified by some form of generalized Dynkin diagrams, irreducible finite dimensional representations are highest weight representations etc...

One major difference is that complete reducibility is lost for finite dimensional representations over $\mathbb{C}$, and the theory shares therefore many similarities with the techniques developed for categories of infinite dimensional complex representations of semisimple Lie algebras (category $\mathcal{O}$) and modular representation categories in positive characteristic.

We will discuss some of the classical topics with an emphasis on the $\mathfrak{gl}(m|n)$-case: Classification of simple Lie superalgebras, algebraic supergroups, Finite-dimensional representations, Character formulas, description of blocks, fusion rules. Along the way we will encounter Deligne's interpolation categories and quivers with relations.

Date and time of the seminar: Will be decided at the meeting.

Prerequisites: Good knowledge of Lie algebras and their representations, some background about categories and functors and algebraic groups.

Organizational meeting: Friday, Jan. 24th, 4.00pm in seminar room 1.008

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