Graduate Seminar on Advanced Algebra

Nichols algebras and generaliations of universal enveloping algebras

Winter term 2021

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There are several ways to generalize the universal enveloping algebra $\mathcal{U}(\mathfrak{g})$ of a Lie algebra $\mathfrak{g}$. One can replace $\mathfrak{g}$ by a more general structure such as a Lie superalgebra, a $\mathbf{Z}/2\mathbf{Z}$-graded variant of a Lie algebra, which admits again a universal enveloping algebra.

Another generalization, mostly due to Drinfeld and Jimbo, is the following: If $\mathfrak{g}$ is a symmetrizable Kac-Moody algebra (or simply a semisimple Lie algebra over $\mathbf{C}$), $\mathcal{U}(\mathfrak{g})$ can be described by generators and relations (the Serre-Chevalley relations). These relations can be deformed depending on a parameter q (or a complex number q). The resulting Hopf algebra $\mathcal{U}_q(\mathfrak{g})$ is an instance of a *quantum group*, a certain non-cocommutative Hopf algebra. Such deformations also exist for Lie superalgebras.

These different generalizations share many features such as PBW bases and triangular decompositions. Nichols algebras provide a common generalization for which one can study such structures. Nichols algebras are by construction Hopf algebra objects in certain braided monoidal categories. This seminar is an introduction to the theory of Nichols algebras. We will study their structure theory and how they generalize the Hopf algebras above. The theory will be accompanied with a strong emphasis on explicit examples.

Date and time of the seminar: Thursdays 4-6pm.

Prerequisites: Good knowledge of Lie algebras and their representations as in the book of Humphreys (in particular Serre-Chevalley relations and root systems and Verma/standard cyclic modules), some background about monoidal categories, basic knowledge about Hopf algebras and their modules/comodules (e.g. Chapters I-III and XI in Kassel's book on quantum groups).

Organizational meeting: Thursday, July 22nd, 4.15pm via Zoom

Literature: Heckenberger, István; Schneider, Hans-Jürgen *Hopf algebras and root systems.* Mathematical Surveys and Monographs 247. xix, 582 p. (2020)

and selected other texts.

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