

# Harmonic Analysis, Problem set 9

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**Due on Friday, 2016-12-23** If you hand in on Thursday, then you can receive the graded homework on Friday.

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**Problem 1.** A  $(1, q)$ -atom is a function  $b$  such that

- (a)  $\text{supp } b \subset B$  for some ball  $B \subset \mathbb{R}^d$ ,
- (b)  $\int b = 0$ ,
- (c)  $\|b\|_q \leq |B|^{-1/q'}$ .

Let  $1 < q \leq \infty$ . Show that for every  $(1, q)$ -atom  $b$  we have

$$\|Db\|_{L^1(S^2)} \leq C_q.$$

Hint: outside of the tent  $T(10B)$  one can use the estimates for the bad part of the CZ decomposition from the lecture notes. Inside this tent one can use the  $L^q(S^2)$  embedding.