EXAM SYLLABUS FOR V5B7: GEOMETRIC ASPECTS OF HARMONIC ANALYSIS

(FEBRUARY 9, 2017, IN OFFICE 3.012)

DIOGO OLIVEIRA E SILVA

(1) What is the Vitali covering lemma? Which implications does it have for the Hardy–Littlewood maximal function? How can you derive the Lebesgue differentiation theorem?
(2) What happens if instead you average with respect to a family of rectangles of unbounded eccentricity? Or arbitrary orientation? Why?
(3) What does the Brunn–Minkowski inequality state? How do you prove it? How does the isoperimetric inequality follow?
(4) What is the Radon transform? How does the Fourier transform interact with the Radon transform? Which bounds for the Radon transform do you know? How are these related to the problem of regularity of sets?
(5) What is a Kakeya set? How do you construct one? What does the Kakeya conjecture state? What about the Kakeya maximal function conjecture? How do estimates for the Radon transform when $d = 2$ relate to the Kakeya problem?
(6) What does the ball multiplier conjecture state? Can you briefly describe Fefferman's solution to the ball multiplier conjecture? How does it connect to the Hilbert transform?
(7) Can you state the principle of non-stationary phase? The principle of stationary phase? Van der Corput's lemma? What is van der Corput's lemma in higher dimensions? What do these estimates imply for the decay of the Fourier transform of a surface-carried measure?
(8) What are spherical maximal averages? Which bounds do you know for it? How are square functions relevant for this problem?
(9) State the Tomas–Stein inequality. What is Knapp's counterexample, and which consequences does it have?
(10) Can you sketch a proof of the restriction conjecture in two dimensions? Why does the restriction conjecture imply the Kakeya conjecture?

Diogo Oliveira e Silva, Hausdorff Center for Mathematics, 53115 Bonn, Germany
E-mail address: dosilva@math.uni-bonn.de

Date: January 24, 2017.