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Functional Analysis and Partial Differential Equations

Sheet Nr.1 Due: 28.10.2016

Exercise 1

Prove that $\mathbb{B}(X)$ with the sup norm is a Banach space.

Exercise 2

Let X be a metric space equipped with the distance $d(\cdot,\cdot)$.
Prove that $C_b(X)$ is a Banach space.

Let $C_0(X) \subset C_b(X)$ be the subset of functions $f \in C_b(X)$ with

$$f(x) \to 0 \text{ for } d(x,x_0) \to \infty.$$

Here $x_0 \in X$. Verify that this notion is independent of x_0 and prove that $C_0(X)$ is a Banach space.

Exercise 3

Prove that the map from l^1 to c_0:

$$l^1 \ni (x_j) \to \left(y \to \sum_{j=1}^{\infty} x_j y_j\right) \in c_0^*$$

is welldefined and an isometric isomorphism. By this we mean that it is injective and surjective, and

$$\|(x_j)\|_{l^1} = \left\| \left(y \to \sum_{j=1}^{\infty} x_j y_j\right) \right\|_{c_0^*}.$$

Exercise 4

Let $\frac{1}{p} + \frac{1}{q} = 1$, $1 \le p,q \le \infty$. Prove that

$$l^q \ni x \to \left(y \to \sum_{j=1}^{\infty} x_j y_j\right) \in (l^p)^*$$

and that this map is an isometry. Is it surjective?