“NONLINEAR FOURIER TRANSFORM”
S4B1: ANALYSIS GRADUATE SEMINAR
(WINTERSEMESTER 2015/16)

1. Introduction

The nonlinear Fourier transform and the nonlinear Fourier series are basic models for a type of analysis done in many parts of mathematics. Related key words are: scattering transform, orthogonal polynomials, Schur's algorithm, Riemann-Hilbert problems, integrable systems, operator theory. This seminar will present an introduction to the theory that runs parallel to the usual basic development of linear Fourier analysis, with particular emphasis on the intricacies in the nonlinear setting that occur on the space of square summable sequences. For interested students, the seminar may lead to a Master thesis assignment. Due to the foundational nature of the subject, this seminar also constitutes an excellent addition for students interested in related fields such as mathematical physics or integrable nonlinear equations.

2. List of topics

From the main source:[1] T. Tao and C. Thiele, *The Nonlinear Fourier Tranform.*

(1) The nonlinear Fourier transform on ℓ^0, ℓ^1 and ℓ^p.
(2) The nonlinear Fourier transform on $\ell^2(\mathbb{N})$.
(3) The nonlinear Fourier transform on $\ell^2(\mathbb{Z})$: the case of bounded a.
(4) The nonlinear Fourier transform on $\ell^2(\mathbb{Z})$: the case of unbounded a.
(5) The Riemann-Hilbert problem for rational functions.

From other sources:

(6) M. Christ, J. Colliander and T. Tao, *Asymptotics, frequency modulation, and low regularity ill-posedness for canonical defocusing equations.* Amer. J. Math. **125** (2003), no. 6, 1235–1293.

Focus on Theorem 2 for NLS, especially sections 2, 3 and 4.

(7) M. Christ and A. Kiselev, *Maximal functions associated to filtrations.* J. Funct. Anal. **179** (2001), no. 2, 409–425.

[1] Available at `http://www.math.ucla.edu/~thiele/lecturenotes/nlft.pdf`.

(8) M. Christ and A. Kiselev, *Absolutely continuous spectrum for one-dimensional Schrdinger operators with slowly decaying potentials: some optimal results.* J. Amer. Math. Soc. **11** (1998), no. 4, 771–797.
Focus on the proof of Theorem 1.1.

(9) R. Coifman and S. Steinerberger, *Nonlinear phase unwinding of functions.* Preprint available at `http://arxiv.org/abs/1508.01241`.

(10) P. Deift and R. Killip, *On the absolutely continuous spectrum of one-dimensional Schrdinger operators with square summable potentials.* Comm. Math. Phys. **203** (1999), no. 2, 341–347.

(11) V. Kovač, *Uniform constants in Hausdorff-Young inequalities for the Cantor group model of the scattering transform.* Proc. Amer. Math. Soc. **140** (2012), no. 3, 915–926.

(12) A. Máté, P. Nevai and V. Totik, *Asymptotics for the ratio of leading coefficients of orthonormal polynomials on the unit circle.* Constr. Approx. **1** (1985), no. 1, 63–69.

(13) C. Muscalu, T. Tao and C. Thiele, *A Carleson theorem for a Cantor group model of the scattering transform.* Nonlinearity **16** (2003), no. 1, 219–246.
Focus on Theorem 1.4, especially sections 3 and 4.

(14) C. Muscalu, T. Tao and C. Thiele, *A counterexample to a multilinear endpoint question of Christ and Kiselev.* Math. Res. Lett. **10** (2003), no. 2-3, 237–246.

Hausdorff Center for Mathematics, 53115 Bonn, Germany
E-mail address: `dosilva@math.uni-bonn.de`
E-mail address: `thiele@math.uni-bonn.de`