V5B2 - Selected Topics in Analysis and PDE
Dispersive PDEs: deterministic and probabilistic perspectives

Summer Term 2025

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Lecture: 10:00 - 12:00 Friday, MATH / SemR 0.008
Topics:

The lecture aims to provide the basics of dispersive partial differential equations (PDEs) from both an analytic point of view and also a probabilistic point of view. Dispersive PDEs are a class of PDEs where different frequencies propagate at different velocities. We shall mainly use Schrödinger equations and wave equations as examples and use tools from harmonic analysis and probability theory to study well-posedness (existence, uniqueness, and stability under perturbation) of these equations.

In the first part of the lecture, we discuss deterministic well-posedness theory of nonlinear Schrödinger equations and nonlinear wave equations. Specifically, we cover

- Local well-posedness via the Banach fixed-point theorem;
- Strichartz estimates for Schrödinger and wave equations;
- Local well-posedness for Schrödinger and wave equations in subcritical Sobolev spaces;
- Global well-posedness for Schrödinger and wave equations;

In the second part of the lecture, we discuss probabilistic well-posedness theory of nonlinear Schrödinger equations and nonlinear wave equations, which goes beyond deterministic well-posedness results. The specific topics include

- Randomized initial data;
- Almost sure local well-posedness in supercritical Sobolev spaces;
- Almost sure global well-posedness.

If time allows, we may discuss stochastic dispersive PDEs.

Prerequisites:
Analysis: basic real analysis; some familiarity in Fourier analysis is helpful.
Probability: Gaussian random variables, independence.

Exam: The first oral exam period is the week of 28 July – 1 August. The second oral exam period is the week of 22 September – 26 September.

References:

- Burak Erdoğan, Nikolaos Tzirakis, *Dispersive Partial Differential Equations. Well-posedness and Applications.*

- Tadahiro Oh, *Nonlinear Schrödinger Equations, Dispersive Equations.* Lecture notes (2018).
- Tadahiro Oh, *Probabilistic Perspectives in Nonlinear Dispersive PDEs.* Lecture notes (2017).
- Terance Tao, *Nonlinear Dispersive Equations: Local and Global Analysis.*
- Monica Vişan, *Oberwolfach Seminar: Dispersive Equations.*