

# Syllabus for Real and Harmonic Analysis

## Key words

Fourier series, basic harmonic analysis, Maximal functions, Birkhoff's pointwise convergence theorem, Calderón–Zygmund theory, Stationary phase, Restriction estimates, Weigthed norm inequalities, Multiplier theory, Littlewood–Paley theory, Pointwise convergence of Fourier series.

## Infrastructure

Course instruction will run April 7 - July 11 (no lectures or tutorials in the last week of the semester). Due to holidays, there are no lectures on Monday April 21 and Thursdays May 1, May 29, and June 19, and no lectures in the week from June 9 to June 13. The final exam will occur July 28 - August 1.

## Lectures

**Lecturers:** Dr. Michel Alexis (alexis@math.uni-bonn.de), Prof. Dr. Asgar Janneshan (ajannesh@math.uni-bonn.de).

Lectures are Mondays and Thursdays, 14:00-16:00 in Room 0.006 of Endenicher Allee 60. The two lecturers will alternate lecturing each week. Every week, handwritten lecture notes will be uploaded to E-campus. *We make no guarantees about the quality of these notes.*

## Tutorials

**Tutor:** Noa Bihlmaier (nobi@math.uni-bonn.de).

Tutorials are Tuesdays 10-12 and Thursdays 16-18, in Room 1.008 of Endenicher Allee 60. Depending on enrollment, we may cancel one of the two tutorial sessions; this will be decided at the end of the second week. The first tutorial will occur Tuesday, April 15. All students should sign up for precisely one of the two tutorials on E-campus.

## Grading

Homework assignments are due weekly during Thursday lectures. The first assignment is due on Thursday April 17. Assignments will be released one week before they are due, and will consist of 4 problems each. Solutions are to be submitted in groups.

Students must earn at least 50% on homeworks to qualify for the final oral exam, occurring July 28-August 1.

## Course Texts

Our main reference will be [6]. However we may also refer to other textbooks, such as [3], [7], [2], as well as some papers such as [8] and [1].

## Schedule

All chapters refer to [6], unless specified otherwise.

Week (lecturer)	Topics/Comments
07.04-11.04 (Michel)	1.1, 1.2, (Bernstein's inequality) 1.4, 1.6
14.04-18.04 (Asgar)	2.3, 2.4 (Lebesgue differentiation theorem), maximal inequalities and pointwise convergence in ergodic theory
21.04-25.04 (Michel)	2.5 (Monday no lecture)
28.04-02.05 (Asgar)	3.3, 3.4, 3.5 (Thursday no lecture)
05.05-09.05 (Michel)	3.5, 3.6, (parts of) 4.1
12.05-16.05 (Asgar)	5.2 (Khinchine's inequality, Lemma 5.5), 9.4 of [4] (an $L^1$ function with divergent Fourier series), misc topics from Chapters 5-6
19.05-23.05 (Michel)	7.1, 7.3.1, 7.3.2, 7.4, 7.5
26.05-30.05 (Asgar)	7.5, 8.1 (Thursday no lecture)
02.06-06.06 (Michel)	8.2, 8.4 (square function estimates, multiplier operators), 4.2 (stationary/nonstationary phase, Fourier transforms of surface measures)
09.06-13.06 (Asgar)	Pentecost (Pfingst) holidays
16.06-20.06 (Michel)	TBD, something in weighted estimates, maybe sparse bounds [5] or remodeling [1] (Thursday no lecture)
23.06-27.06 (Asgar)	11.1, 11.2
30.06-04.07 (Michel)	continued weighted inequalities, last HW assignment goes out
07.07-11.07 (Asgar)	Walsh Carleson [8], last HW assignment due
14.07-18.07	lectures canceled - study period
21.07-25.07	study period
28.07-01.08	Final oral exams

## References

- [1] Domelevo, Komla, et al. "The matrix  $A_2$  conjecture fails, ie  $3/2 > 1$ ." arXiv preprint arXiv:2402.06961 (2024).

- [2] Duoandikoetxea, Javier. Fourier analysis. Vol. 29. American Mathematical Society, 2024.
- [3] Grafakos, Loukas. Classical Fourier Analysis. Vol. 2. New York: Springer, 2008.
- [4] Kahane, Jean-Pierre. Some random series of functions. Vol. 5. Cambridge University Press, 1985.
- [5] Lerner, Andrei K., and Fedor Nazarov. "Intuitive dyadic calculus: the basics." *Expositiones Mathematicae* 37.3 (2019): 225-265.
- [6] Muscalu, Camil, and Wilhelm Schlag. Classical and Multilinear Harmonic Analysis: Volume 1. Vol. 137. Cambridge University Press, 2013.
- [7] Stein, Elias M., and Timothy S. Murphy. Harmonic analysis: real-variable methods, orthogonality, and oscillatory integrals. Vol. 3. Princeton University Press, 1993.
- [8] Thiele, Christoph. "The quartile operator and pointwise convergence of Walsh series." *Transactions of the American Mathematical Society* 352.12 (2000): 5745-5766.