
Partial Differential Equations and Modelling

Sheet Nr.7

Due: 16.06.2017

Exercise 1

Let μ be a probability measure on $[-1, 1]$. Let $f \in C([-1, 1])$. Prove that f is cyclic for the multiplication by x if and only if $x \in \text{supp } f$ whenever $f \in \text{supp } \mu$.

Exercise 2

Find an explicit formula for the solution $\psi : \mathbb{R} \times \mathbb{R}^d \mapsto \mathbb{C}$ to

$$i\psi_t + \Delta\psi = 0, \quad \psi(0, x) = (2\pi)^{-\frac{d}{2}} e^{-|x|^2/2}, \quad x \in \mathbb{R}^d.$$

Hint: Apply the Fourier transform.

Exercise 3

Prove the general uncertainty relation

$$2\|(x - x_0)f\|_{L^2}\|(\xi - \xi_0)\hat{f}\|_{L^2} \geq \|f\|_{L^2}^2$$

and characterize functions f for which the equality holds.

Exercise 4

Let $H = L^2(0, 1)$ and let $T = i\partial_x$ with domain $C_0^\infty(0, 1)$. Find a selfadjoint extension.