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Summer Term 2017

Partial Differential Equations and Modelling

Sheet Nr.6

Exercise 1

Let $H = L^2(0,1)$, $D(T) = C_0^\infty(0,1)$ the compactly supported functions on $(0,1)$, and $Tf = -f''$.

   a) Determine T^*.

   b) Determine all selfadjoint operators S with $T \subset S \subset T^*$.

Exercise 2

Let T be a densely defined selfadjoint operator on H and $A \in L(H)$ selfadjoint. Prove that $T + A$ is selfadjoint.

Exercise 3

Let (T_j) $1 \leq j \leq d$ be densely defined selfadjoint operators on H. We say that the (T_j) commute if their Cayley transforms and their adjoints commute. Prove the following statements:

   a) T_1 and T_2 commute iff their generated groups S_{T_1} and S_{T_2} commute, i.e.

$$S_{T_1}(s)S_{T_2}(t) = S_{T_2}(t)S_{T_1}(s), \qquad \text{for all } s, t \in \mathbb{R}.$$

   b) If (T_j) $1 \leq j \leq d$ commute, then there exists a Radon measure μ on $\mathbb{R}^d \times \mathbb{N}$ and a unitary operator $U : L^2(\mu) \to H$ so that

$$U(x_j f) = T_j U f$$

   for all $f \in D(M_{x_j})$ and $1 \leq j \leq d$.

Exercise 4

Suppose that $a \in \mathcal{S}(\mathbb{R}^{2d})$ has real values. Prove that $a^w(x, D)$ is selfadjoint on $L^2(\mathbb{R}^d)$.

Matheparty Information:
Die Fachschaft Mathematik feiert am 1.6. ihre Matheparty in der N8schicht. Der VVK findet am Mo. 29.05., Di. 30.05. und Mi 31.05. in der Mensa Poppelsdorf statt. Alle weitere Infos auch auf fsmath.uni-bonn.de.

The student council of mathematics will organize the math party on 01/06 in N8schicht. The presale will be held on Mon 29/05, Tue 30/05 and Wed 31/05 in the mensa Poppelsdorf. Further information can be found at fsmath.uni-bonn.de <http://fsmath.uni-bonn.de/>.