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Alternative claims to the discovery of modern logic: Coincidences and diversification

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1 Multiple and alternative discoveries

No discipline could lay stronger claim to clarity and unequivocality than logic. Curiously, though, the historical genesis of modern logic presents a picture riven with rival claims over the discipline's founding contributions. As will be shown, protagonists from highly different backgrounds assert that the genesis of modern logic—indeed, its very discovery—rests on their contribution. Not only are these claims, to some extent, mutually exclusive, they also cut across standards of scientificity and rationality. By standard narratives of the history of modern logic, some claimants to paternity seem downright obscure and anti-rational. Yet, all these claims are made within a narrow time-frame, and they all refer back to the same developments in mathematics. This makes it impossible to easily dismiss the rival claims.

This paper argues that the co-existence of alternative claims concerning the discovery of modern logic, in fact, places the historian in an advantageous situation. It might appear as if the alternative claims make the the genesis of modern logic into a story of a *multiple discovery*, the coincidence of simultaneous discoveries of the same phenomenon. This is, however, not what the historical picture shows. The competing claims, very explicitly placed by the protagonists themselves, rather have to be viewed as stages within an open-ended process that would contribute not to an integrated picture of logic, but generate diversified conceptions of rationality. It is

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precisely the differences between the positions that tended to be emphasized. Fundamental notions, seemingly incontestable today, were still very much in dispute. These disputes as to the discovery of modern logic can serve to both enrich our understanding of the historical milieu in which logic emerges as a modern discipline and, perhaps more importantly, question extant normative standards used to reconstruct the historical genesis of logic.

Multiple discoveries are, at least since Thomas Kuhn (1959) and Robert K. Merton (1961, 1963), a well-established topos in the historical and philosophical reconstruction of scientific discoveries. They are usually viewed as two (or more) distinct and independent discoveries of the same result (which may pertain to any stock element of science, such as theories, laws, relevant observations etc.). In many cases—the standard example being the multiple discovery of the principle of energy conservation—these discoveries are later integrated into a comprehensive theory, thereby revealing the various contributions as being only partial discoveries. Serious problems, however, hamper this account: in most cases, it is very difficult to determine whether or not the discoveries are really totally independent. Can it really be the case that two researchers working within the same period can be considered completely independent? Likewise, it is hard to establish the identity of the discovery. What does it mean for two discoveries that are made in different contexts to be identical in all relevant aspects? Kuhn posed this question in his paradigmatic paper on the discovery of energy conservation without, however, exhausting its implications in detail. Strictly speaking, a simultaneous discovery would presuppose that different protagonists announced "the same thing at the same time" but that is not what one actually finds (Kuhn, 1959, p. 70). Far from being a well-orchestrated set of simultaneous discoveries, the historical process of discovery can rather be seen as a series of partial events that require further integration and become identified or unified only in hindsight.

The situation changes in important respects when switching to the scenario that shall be dubbed "alternative discoveries". These are cases where, starting from what may appear as a multiple discovery, different scientists explicitly claim to be the first, or the genuine, discoverer of a relevant result. The multiplicity is thus seen and stated not retrospectively by historians, but by the protagonists themselves. What sets cases of alternative discovery apart from multiples, in particular, is the aim behind stating the multiple discovery. When a scientist actually claims to have contributed to a multiple discovery, this may be just a pitch for priority. In many cases, however, a different goal is pursued: the alternative claims tend to emphasize precisely the uniqueness, the singular value of the various contributions, respectively, and thus focus on the differences that lie behind seemingly very similar or

even identical discoveries. This is especially revealing in disputes concerning conceptual aspects of the various discoveries. Lines of differentiation are, for instance, grasping the true significance of a discovery, giving it its proper place within a larger theory, embedding it in a general context, giving the best proof, showing the most salient application. Alternative discoveries, in this sense, stand between the identity typically generated in cases of multiple discoveries, and the emphasis on differences between the various partial discoveries. This emphasis also reveals where the protagonists themselves located the most relevant innovations.

The alternative claims regarding the discovery of modern logic cannot be integrated smoothly into one great multiple discovery. That logic is, in any case, intimately related with rationality and with the foundations of "science" (in the broad sense) remains uncontroversial. Yet, on the alternative discoveries' reading, differences become visible that reveal that even in fundamental questions of modern logic the bandwidth of possible options was far broader than one might expect. Since logic pertains to foundational ideas, this suggests that even among research traditions that share a common ideal of scientific rationality, this ideal could be understood in rather different ways. The relevant question, then, is not: have the actors involved indeed made the very same discovery but, rather, where do the relevant lines of agreement and disagreement lie and to what extent can this mapping inform our understanding of fundamental notions at play in the 'official' account of the genesis of modern logic?¹

2 Alternative claims: contesting the discovery of modern logic

Around 1900, authors from highly different, even—from today's perspective absolutely irreconcilable traditions lay claim to the discovery of modern logic. The following features are taken to be defining characteristics of 'modern' logic: the new logic is based on mathematics and takes novel developments in 19th mathematics into account; it incorporates a theory of relations and thus goes beyond traditional syllogistics; it aims at developing a type of science that occupies the *most fundamental* place in the system of scientific disciplines.

Yet, these alternative claims have not become part of the common historiography of logic. Here, typically, a mathematical-philosophical line is

¹Pulkkinen (2005) illustrates the difficulties inherent in singling out an 'official' line of the development of logic in the 19th century. Peckhaus's highly critical review (Peckhaus, 1997) on the one hand draws a yet richer picture, while on the other hand trying to clarify the situation further. For an appreciative account of some of the alternative lines in the history of logic, cf. also Bowne (1966); Hansen (2000); Haaparanta (2009); the latter is a broad overview still based on the official history of logic. For a case study concerning multiple discoveries or re-discoveries in the field of logic, cf. Schlimm (2011).

singled out and has become the 'official' lineage, with well-known and generally respected protagonists such as de Morgan, Boole, Peano, Frege, Russell or Whitehead, and with a number of lesser known, but nonetheless equally accepted players such as Louis Couturat. Other authors, associated with more traditional philosophical projects (such as Sigwart, Wundt or Lotze), were largely eclipsed by changes within the field of philosophy, and by the increasing emphasis on the role of mathematics for modern logic.² The rivals who laid alternative claims as to the discovery of modern logic are known neither as possible claimants for the title of an inventor of modern logic, nor are they considered part of the official history of philosophy; they are simply banished to obscurity. Perhaps the most prominent claimants in this context are Wilhelm Ostwald (1853–1932), the Nobel Prize winning Leipzig chemist, founder of the discipline of physical chemistry and popularizer of a "monistic" world-view, and Hans Driesch (1867–1941), pupil of the equally famous and infamous zoologist and Weltanschauungs-thinker Ernst Haeckel, an embryologist of the first rank whose philosophical ideas on irreducibly a-mechanistic "entelechies" in living organisms were presented by Vienna Circle members as the very paradigm for anti-scientific metaphysics $(Carnap et al., 1929, p. 312).^3$

Although the recent literature has devoted considerable attention to the mutual influences and points of intersection between different traditions in 19th and 20th century philosophy such as neo-Kantianism, phenomenology and analytical philosophy,⁴ the emergent picture remains pointillist; it focuses on individual terms or concepts and excludes awkward alternative thinkers such as Ostwald or Driesch. Yet, the lines separating these more radically alternative traditions from 'official' analytical philosophy are by no means sharp: Ostwald and Russell pursue at least one common goal by publishing, in German and English, respectively, Wittgenstein's *Tractatus*; Ostwald and Couturat join forces in propagating new artificial languages (such as Esperanto or, preferred by Ostwald and Couturat, "Ido") for daily use (cf. Ziche, 2009). And in one of the early classics of analytical phi-

 $^{^2 {\}rm Yet}$ other players such as Husserl and Peirce are perhaps even more difficult to place; they will not be discussed here.

 $^{^{3}}$ On Driesch as biologist, cf., e.g., Jahn (1998, pp. 444–445); on Ostwald, cf., e.g., Görs et al. (2005).

⁴Important contributions come from Gottfried Gabriel, Hans Sluga, Michael Friedman and Alan Richardson. An example for the pointillist character of many studies of the interactions between the fields is Sluga (1997). When discussing Frege's indebtedness to the Neo-Kantians that is visible in his use of the term "Wahrheitswert" (a term playing a considerable role also in the "significal" debates discussed in this paper in §§III and IV), Sluga describes the relationship between Frege and his Neo-Kantian contemporaries in terms of a "quotation", a "borrowing", a "link" and of "connections", but stresses that "[i]t is precisely in the nature of such borrowings that they are partial" (Sluga, 1997, pp. 31–32).

losophy, Carnap's *Logischer Aufbau* from 1928, Ostwald and Driesch stand peacefully next to Russell and Whitehead (Carnap, 1928, pp. 3-4).⁵

Ostwald bases his scientific world-view on one single principle—in other words, he argues vigorously for a 'monism'—, namely the principle of energy conservation. Although he duly celebrates the innovative character of this principle and the heroic achievements of its 19th-century discoverers, he replaces it in its function as the most general principle in science by another principle that he claims to have introduced himself. When rewriting his Lectures on the Philosophy of Nature from 1902 for a second edition appearing in 1914, he states that it was him, Ostwald, who had discovered (with the disclaimer "to the best of my knowledge" whose modesty appears, given his repeated statements to the same effect, as rather tongue-in-cheek) that a great progress in the philosophy of science can be achieved by acknowledging "logic as the first and most general science ['Wissenschaft'], even more fundamental than mathematics" (Ostwald, 1914, VI). Already in 1909 he had stated, referring in very concrete terms to the mathematical theories forming the inspiration for his claim, that he had made the important discovery "that logic, which better and more generally should be called the theory of manifolds, is an even more general science than mathematics" (Ostwald, 1909).⁶ With his reference to theories of manifolds, he refers to a mathematical concept that had, in different meanings but always with highly innovative results, been used by Hermann Grassmann in his multidimensional algebra, by Bernhard Riemann in differential geometry and by Georg Cantor in his set theory, and that had provided concepts that proved indispensable in the genesis of modern logic. An admirable overview over these theories is given in Whitehead's first major publication, his *Treatise* on universal algebra with applications (Whitehead, 1898).

What is the basis of these claims as regards logic? Ostwald links his interest in mathematics and logic to the problem of concept formation and the task of constructing new types of languages for the purpose of absolutely clear communication. Without mentioning Frege,⁷ he adopts the term "Be-griffsschrift" for his own program of establishing one (or several) languages that aspire to complete definiteness. For him, the new logic devolves from the attempt to generalize one of the central tasks of science, that of clarifying

⁵Driesch is mentioned here, under the title "Konstitutionstheorie"—a term that Carnap employs to characterize his own project—as the author of one of the most important modern "Begriffssysteme"; Ostwald is seen as a more distant point of reference because Ostwald (just as the psychologist-philosophers Wundt and Oswald Külpe or the theologian Paul Tillich) gives a classification of forms of science, i.e., of systems of concepts, but no derivation of the concepts employed in science.

⁶On the broader context, cf. Ziche (2006, 2008).

⁷It seems clear that Ostwald came into contact with Fregean ideas in the context of publishing Wittgenstein's *Tractatus*; however, there seems to be no explicit reference to Frege's work in Ostwald's writings.

our concepts so as to arrive at well-defined concepts. Just as in Helmholtz's *Thatsachen in der Wahrnehmung*, and in the principle of energy conservation, this procedure is substantiated by the fundamental role played by invariants and relations in science (von Helmholtz, 1878).⁸ In new algebraic theories, such as the theory of groups or of manifolds, Ostwald discerns a general framework for thinking order and structure in terms that can be applied to concept formation.

The same terms are featured in Hans Driesch's "Ordnungslehre". Driesch views his "Ordnungslehre" explicitly as "logic, understood in the broadest sense" (Driesch, 1923, p. 2).⁹ He does not talk in terms of a discovery of modern logic, but in terms of giving logic its proper place within the system of various sciences. His "Ordnungslehre" aims to rival the status of logic as the most fundamental science: "Ordnungslehre" is "die wahre erste Philosophie", the genuine first philosophy (Driesch, 1913, V). Interesting enough, he mentions in one and the same paragraph Ostwald (critically, because he does not really penetrate to the level of genuine metaphysics) and, more approvingly, Moritz Schlick (because he at least makes definite statements regarding the inexistence of an empirical reality) (Driesch, 1926, p. 8).

Some common patterns, and some marked differences between the different origin stories of logic, stand out immediately. All protagonists, regardless of whether they had a formal education in mathematics or not, looked at very much the same theories within mathematics to substantiate their claim that a "new logic" was both required and possible. The two most important examples are new algebraic theories such as group theory, generalizing on the usual forms of algebraic operations, and the theory of manifolds as a natural extension of traditional theories within arithmetic and geometry. These developments are intimately linked with the re-assessment of the seemingly most obvious foundations and fundamental operations of mathematics. A particularly prominent example for the interplay between highly innovative achievements and very basic topics is a new interest in the

⁸On the history of the logic of relations, cf., e.g., Merrill (1990); on invariants as a mathematico-philosophical issue, cf. Ihmig (1997).

⁹His "Ordnungslehre" starts with a—possibly solipsistic—immanentism, and is intended to prepare the ground for any future metaphysics. The fact that I experience something that is ordered in a particular way ("Erleben von bestimmtem Geordneten", Driesch, 1923, p. 1) forms the basic topic of philosophy. References to recent results in mathematics are to be found, e.g., in the chapter on "manifold" (p. 136) where Driesch refers to Riemann—but only in order to distinguish his concept of manifold from the corresponding concept in mathematics. Driesch also employs his concept of "order" in his works on theoretical biology, e.g., in the programmatic treatise (Driesch, 1924), and throughout his Gifford lectures on the philosophy of the organic world (Driesch, 1909). Cf. also Vollenhoven (1921).

justification of the complex numbers.¹⁰ Here, however, the participant actors already part company: shared interest in these mathematical theories and a shared agenda to develop new and more precise forms of language no longer produce the same conclusions. The cases of Ostwald and Driesch indicate that, in taking up inspirations from recent trends in mathematics, one could perfectly well focus on very simple cases of application (such as complex numbers) and still think that one had incorporated the most essential aspects of these mathematical innovations. Formal language, then, becomes dispensable.

It is also striking that the typical career paths of the protagonists involve a high degree of switching between fields and disciplines, be it mathematics and philosophy, or, in the case of Ostwald and Driesch, philosophy and the natural sciences. This makes sense in a context where the stated task is disciplinary innovation on both ends of the spectrum of practice: at the level of maximum generality and at the level of highest speciality. Driesch and Ostwald were, in one respect, the most radical among the group of discoverers of logic: they opted for a change of the name of the fundamental discipline they aimed at, based precisely on the new achievements within mathematics, suggesting terms such as "Mannigfaltigkeitslehre" or "Ordnungslehre" for the new discipline. Debates on logic thus fit into the heyday of creating new disciplines; "Gegenstandstheorie" (Meinong, but also Driesch), "Phänomenologie", but the same holds for "Ordnungslehre"—which then should be compared with the role the concept of order comes to play in, for instance, Whiteheadian philosophy of nature.

3 Synthesis in *Synthese*: Pluralistic origins of a logico-philosophical journal

The claims made by Ostwald and Driesch are more than just exotic (or quixotic) hobby-horses of German philosophers of nature. Precisely the same discursive formations can be found in some of the most relevant philosophical journals of this time, and it is in these journals that the rather abrupt changes in conceptions of rationality show most clearly. The very form of these publications typically combines programmatic statements with a diversity of contributions that normally cover a considerably broader range. Even journals that function as the mouthpiece of a rather well-

¹⁰"New" is here taken in the sense that in many publications from highly divergent fields (mathematics, logic, literature, philosophy...) from the years around 1900 one finds a strong tendency to view the complex numbers (despite older discussions in Gauss or Riemann, among others, and despite the well-established use made of complex numbers in the natural sciences) as "paradoxical" or "contradictory". Examples are, among philosophers and philosopher-mathematicians, Husserl (1901, p. 433) and Natorp (1923, p. 239), among mathematicians and logicians, Hankel (1867, V). Cf. Ziche (2008, Chapter VI.3).

defined group of authors typically present a high degree of internal diversity. The alternative traditions in the discovery of modern logic figure alongside each other in one of the first issues of a journal that was to become one of the central forums for modern analytical philosophy, *Synthese*. Here we find rather extensive and affirmative discussions of the role of Driesch as philosopher but also a marked interest in the ideas of Moritz Schlick as well as a whole series of articles by the ex-theosophist Matthieu Schoenmakers.¹¹ Similar observations can be made with regard to other journals such as *Erkenntnis* or *Mind*.¹²

The early history of Synthese offers a telling case study for present purposes. Synthese started as the official organ of the "signifische kring", a group of Dutch intellectuals that perfectly illustrates the problems encountered in the attempt to give a unilinear account of the genesis of modern logic or of analytical philosophy. The movement of "significs", based on the philosophy of language of Lady Victoria Welby, was a predominantly Dutch affair that attracted thinkers with rather different backgrounds. Prominent members of this movement were the writer Frederik van Eeden, the mathematician and philosopher Gerrit Mannoury, the great mathematician and logician Brouwer and the linguist Jacob van Ginneken.¹³ The official "Introduction" to Synthese takes up basic ideas of this group: the journal aims to express the spirit of the day, "de geest van onzen tijd", which was conceived as a "period of synthesis". A deeply and widely felt longing for a synthesis had been unleashed by the increasing specialization and fragmentation that typified the 19th century. A broad alliance is formed, across schools, traditions and disciplines. In the eyes of the editorial board of Synthese, "eminent biologists" fight together with Hegelians for the unity of analysis and synthesis. Categories from the philosophy of the human sciences such as "gelden", i.e., "to have a value", are employed next to motives from gestalt psychology, and it would be hard to conjure up a more diversified list of reference authors than those cited by the editors of *Synthese*: the psycho-physicist Fechner and the physicists Jeans and Einstein, the latter for contributing to a "crisis der zekerheden", a "crisis of certainty"; the mathematicianphilosophers Henri Poincaré and the more heterodox Émile Boutroux (his

¹¹Cf., e.g., his articles on "Zinzeggende beeldspraak" (roughly: a form of image-based or image-directed language that expresses the essence of reality), with the thesis that "Ook de exacte formule is beeldsprak" ("the exact formula, too, is figurative speech") (Schoenmaekers, 1936, p. 7). On Schoenmaekers, cf. de Jager (1992).

 $^{^{12}}$ On the early history of *Erkenntnis*, cf. Hegselmann and Siegwart (1991); they show how *Erkenntnis* emerged from a strong neo-Kantian background.

¹³The history of the significal movement cannot be presented here; a comprehensive overview is to be found in Schmitz (1990). Cf. also the extensive commentary on one of the early texts of the significs-movement (van Eeden, 1897). For the broader context and forms of institutionalization, cf. Heijerman and van den Hoven (1986). On Brouwer and the significal movement, cf. van Dalen (1999, esp. pp. 243–250, 255–270, 367–375).

writings include texts on mathematics and mysticism, and on the philosophy of German Idealism); the idealist Jean-Marie Guyau and the pessimistic and/or metaphysical authors Schopenhauer, van Hartmann, Nietzsche and Bergson. Equally diversified is the list of psychologists/physiologists that *Synthese* references, including von Monakow, Stern, van Maeder, Jung and Janet (Godefroy et al., 1936b).

The motive of a crisis remains prevalent and leads to a whole range of strongly negative statements: "our time is in essence anti-materialist, antiintellectualist, anti-mechanistic". Synthese, however, does not recommend a sceptical or pessimistic metaphysics but aims at integrating all these tendencies into a larger unity modelled on the scientific mode of thinking. This, in turn, is described as "an orientation towards a clearer and more powerful unity" ("oriëntering naar klaarder en krachtiger eenheid"; Godefroy et al., 1936b, p. 6). This formulation shows how different sets of categories coexist at the time in one and the same journal. "Clarity" is an ideal that stands central also in the early manifestos of the Vienna Circle and of analytical philosophy, the terms "powerful" or "forceful" thinking, however, sooner evoke the ideas of metaphysically minded authors such as Nietzsche, Schopenhauer or von Hartmann.

Most revealing is the way in which the significal authors treat the Vienna Circle in Synthese. Much space is devoted to Moritz Schlick, in particular.¹⁴ Already in the first volume of Synthese, Schlick is viewed as a thinker who brings several central issues and problems of current-day thinking into sharp focus (Godefroy et al., 1936a, p. 108). The journal quotes extensively from his writings, and at least one passage expresses strong approval: "the essential and highly pleasing aspect, the living modernity in Schlick is his spiritual drive ['geestesdrang'] towards a synthesis of philosophy and the special sciences ['vakwetenschap']" (Godefroy et al., 1936a, p. 112). In later publications, for instance in an obituary devoted to Schlick in 1936, Schlick is characterized even more explicitly in world-view terminology. He is celebrated as a philosopher who reflects on the "essence" ("zin") of his own time, and who thereby tries to bring about an integration of all areas of knowledge, a unity of science based on strictly scientific principles (Godefroy et al., 1936c, p. 195). This unity also implies a unity of the modes of expression in the sciences. When the editorial board of Synthese states that "what our times desperately needs ['broodnodig'] are clear and transparent concepts" (Godefroy et al., 1936c, p. 197), they state both their own and Schlick's goal. Again, they combine categories and forms of expression belonging to a Weltanschauungs-discourse with those stemming from the official tradition of analytical philosophy.

¹⁴After the war, Otto Neurath (who also was a member of the "Internationale signifische studiegroep", cf. Schmitz, 1990, p. 18) publishes a programmatic overview over the unity-of-science-movement (Neurath, 1946).

Still, there are marked differences between the Vienna Circle and the significs, and these are rooted in fundamental tenets of their respective beliefs. These are clearly formulated in an article that discusses the contrast between "Significa" and the Vienna Circle (Godefroy et al., 1936d). Here, the program of bringing "clarity" into our diffuse and chaotic ("vertroebeld") emotional and intellectual life again serves, from the very first sentence on, as the leitmotif. Language is identified as the cause of this confusion. Yet, the Dutch and the Viennese approach to language differ: in Vienna-style thinking, it is the language of science and of philosophy that comes under scrutiny, whereas in the Netherlands, it is ordinary everyday language that the significal thinkers scrutinize (Godefroy et al., 1936d, p. 332). Again, however, in concrete practice there is a close agreement. The Dutch thinkers propose to work on a large dictionary/encyclopedia-project that should capture the fundamental words ("grondwoorden") necessary for any successful use of language. On the basis of such a fundament, the different levels of language ("taaltrappen", literally: steps of language) could be reconstructed that mediate between the fundamentals of everyday language and the use of language in logic, mathematics or science.

The idea that there are fundamentally different forms of language, related via a whole series of steps, is central for the significs' approach. The focus on language, however, leads to rather sharp divergences between the groups of authors represented in Synthese. This becomes particularly clear in another joint issue that interests both the significal authors and the Vienna Circle: the adequate way of dealing with "Scheinprobleme", i.e., with pseudo-problems in science and philosophy. One way of putting the latter problem distinguishes between "formal-logical" and "immediate-intuitive" uses of language as two extremes that, nevertheless, belong to a unity of language that is to become the subject of significal research (Godefroy et al., 1936d, p. 336). The intuitive, and thereby also the affective and emotional aspects of language are a fundamental and irreducible element of language; "significa comprises more than just a critique of language, or even a synthesis of language; significs has to be directed towards a deeper understanding in the connection between words and contents of the soul ['zielsinhoud']" (Godefroy et al., 1936d, p. 336–337). Still, Synthese aims at a grand synthesis, based on significs and on the results of the Vienna Circle—more precisely, on the "experiences" made by them—with the intention of bringing "some order into the chaos of the thinking of today". That the editors of Synthese could hold that such an approach, based on approaches they themselves took to be highly diversified, could establish order and clarity shows that the concept of clarity itself is far from clear (Godefroy et al., 1936d, p. 338).

During and after the Second World War, some more explicit statements are formulated with regard to central ideas of the thinkers of the Vienna circle, beginning with the question of "Scheinprobleme" and amounting to a differentiation of uses of logic. Meaningless expressions can occur in metaphysics, "[b]ut we do hold the opinion that a logical analysis does not suffice to brand as pseudo-questions, meaningless questions and 'isolated sentences' the many questions appearing in diverse 'doctrines regarding views of life'" (Clay et al., 1946a, p. 22).¹⁵ Again, a highly interesting and highly diversified list of authors is quoted as supporting the significs' investigations into the foundations of knowledge: "Husserl, Mach, Hans Vaihinger, Driesch, Carnap, Schlick" (Clay et al., 1946a, p. 22).¹⁶

The inclusive attitude lasts surprisingly long. The introduction to the post-war issue from 1946 likewise stresses the conciliatory character: "we tried to avoid going to extremes" (Clay et al., 1946b, p. 9), steering a middle way between the forms of "word-thinking" predominant "in the off-shoots of Neo-Kantianism, in Phenomenology, in 'Existenz-philosophie' and in some forms of Neo-Vitalism" on the one hand and the "extreme axiomatic tendencies" on the other. In 1959, *Synthese* announces "changes in our editorial policy" that aim at still greater multidisciplinarity, and warning that the methodology of the social sciences "should beware of the pseudoexactness resulting from a blind imitation of mathematical and physical procedures" (Esser et al., 1959). It is not before 1966, when Jaakko Hintikka takes up responsibility for *Synthese*, that the official statements of the editors provide a more determined attempt to clarify the role of logic in *Synthese*, and to make the "philosophy of science" the defining concept for *Synthese*'s program (Hintikka, 1966).

4 Formalization: From Meccano to mathematics

It has already become evident that, among the highly diversified group of thinkers that has been associated with the development of modern logic, *formalization* was not unequivocally seen as one of the most fundamental tasks. As an alternative, one could also opt for the task of giving each concept its proper place within a larger continuum of types of language. This attitude also marks the personal interest that Brouwer, absolutely the most outstanding logician in the significs' circle, took in this movement. For him, significs had to fulfil the centrally important function of clarifying the affective dimensions of language. He, too, seems to think in terms of a continuum in which these needs are linked to his logical and mathematical

 $^{^{15}}$ Cf. also Groot (1937).

¹⁶Cf. also Godefroy et al. (1939), where Driesch's "Ordnungslehre" is described as "both a real ['zakelijke'] and a formal logic" (p. 459).

endeavours.¹⁷ Interestingly, one can find in the group of authors discussed so far attempts to give some kind of formalization even to this apparently strictly anti-formal attitude, thereby demonstrating why it is difficult to ascribe a clearly defined role to formalization.

One of the clearest confessions of faith regarding the role of logic in the early volumes of *Synthese* is to be found in 1946 in an "Introduction to an introduction": one of the greatest achievements of "the last few decades" is "a deepening of logic, not of academic logic with its fixed system of syllogisms, which has after all received a heavy blow from Brouwer's intuitionism, but logic reborn and renewed from the spirit of mathematics and with which the names of Frege and Russell may be said to be indissolubly connected". Evidently, a broad range of new discoveries was deemed relevant (Clay et al., 1946c, p. 102). But the very same text warns against falling into extremes, and argues against too extreme forms of formalization, the "disease of mathematicians", in a quote from Hermann Weyl that was possibly transmitted via a text by Schlick.¹⁸ Formalization thus enters into a discourse that also concerns the question of whether it is legitimate to abstract from the more intuitive aspects of cognition. In the early *Synthese* a rather critical attitude prevails with respect to formalization.¹⁹

Particularly telling is the way in which Gerrit Mannoury (1867–1956) an autodidact who ended up as professor of mathematics at Amsterdam both criticized and made use of formalization.²⁰ In a short reply to a lecture by Donald C. Williams (1937), Mannoury takes a sharply critical stance towards formalization: "it is impossible to find a formal delimitation for the human activity called science and [...] trying to do so must be regarded as unscientific" (Mannoury, 1937a, p. 369). His argument lies in a critique, not

 $^{^{17}}$ Cf. Brouwer's "specific declaration" in (Brouwer, 1939, p. 9); for Brouwer, according to this document, the role of significa does not so much consist in "taalkritiek", a critical study of language, but rather in "detecting the affective elements which form the basis of the function of words", and in the "creation of a new vocabulary" that makes it possible to communicate over the spiritual tendencies of life.

¹⁸Interestingly, the original quote establishes a connection with yet more distant philosophers: "Wer freilich in logischen Dingen nur formalisieren, nicht sehen will—und das Formalisieren is[t] ja die Mathematikerkrankheit—wird weder bei Husserl noch bei Fichte auf seine Rechnung kommen" (Clay et al., 1946c, p. 104). The quote is from Weyl (1919); Schlick himself quotes this passage in (Schlick, 1926, p. 175).

¹⁹Detlefsen (2005, p. 237) reconstructs the history of formalism according to these lines: "they moved towards a conception of rigor that emphasized abstraction from rather than immersion in intuition and meaning".

²⁰Gerrit Mannoury started his career as "privaatdocent" for the logical foundations in mathematics in Amsterdam and became then extraordinary professor in mathematics and later ordinarius first for "meetkunde, werktuigkunde en de wijsbegeeerte der wiskunde", then for "meetkunde, mechanica en wijsbegeerte der wiskunde" in Amsterdam.²¹ For a more detailed account of the role of formalization within the significal movement, the contributions by Brouwer (1937) are of primary importance.

spelled out in this short statement, of the tenet that ideas may be "chained up to words".

What is needed is a more flexible analysis of language. Mannoury presents such an account in a programmatic paper on the significal foundations of mathematics (Mannoury, 1934), his contribution to the Paris conference on unity of science. He draws a distinction, in close parallelism with the two levels of language already mentioned, between two types of speech acts (hereby explicitly employing the terminology of "speech acts"). For him, there exist indicative, i.e., object-directed, and volitional or imperative modes of speech, and thus it becomes immediately evident that a labelling of ideas by words is based on a far too simplistic view of language. As a means for investigating language he suggests a thorough study of the development of language in children and explicitly quotes Brouwer's studies in the philosophy of language as an illustration of his own ideas. The distinguishing feature of mathematics is seen in its universal relationalism: mathematics works exclusively on the level of relations; mathematical definitions relate concepts or symbols without asking whether they correspond to any kind of reality. As a remedy, Mannoury asks for a well-defined psychological foundation without which any kind of distinction—and, one is supposed to add, any talk in terms of relations—is bound to remain "leeres Gerede", empty chatter. Mannoury here takes psychology in an extremely broad sense that again jumps over seemingly deep chasms between different traditions when citing Reichenbach, Russell, Weyl, but also Hugo Dingler, as relevant authors contributing to the kind of psychology he aims at. This extreme extension of the idea of psychology might offer the only way to bypass the rather obvious charge of psychologism; if psychology itself can be linked to the endeavour to provide a basic, fundamental discipline that lies beyond the traditional distinctions between, e.g., mathematical and empirical disciplines, the charges of psychologism lose force.²²

Mannoury then sketches in some detail a systematical approach to language that he deems compatible with his basic view of language, but that, on the other hand, displays some typical elements of formal languages. Again, an important innovation from logic is taken up, namely the role of relations that have to supplement the elements of language or thought that are detected via introspection: "'Elements' and 'relations' are, therefore,

 $^{^{22}}$ In all these aspects, there are highly interesting links with the school of introspective thought psychology in Würzburg; protagonists of this school aimed at making this form of psychology into a genuine experimental science and also reflected on new forms of science (Oswald Külpe, mentioned above, is an example). Karl Bühler, also a member of the Würzburg school, did pioneering studies in child psychology that might be related to the significs' idea of investigating the language of children. On the 'Würzburger Schule', cf. Ziche (1999); Kusch (1999).

the necessary building blocks on the basis of which the whole terminology has to be developed, in precisely the way the mathematician constructs a set theoretical or combinatorial-topological terminology" (Mannoury, 1937b, p. 186–187). Mannoury's criterion for a successful reconstruction of language is the "Anschluß", the compatibility with the "living languages", and this criterion is understood as explicitly non-formal. In some respects, in particular in Mannoury's distinction between an "Es-Sprache" and an "Ich-Sprache", we can detect strong links with Carnap's investigations into a possible physicalisation of the language of psychology that were published in 1932/33, also in *Erkenntnis* (Carnap, 1933).

But what can then be the use, in a setting that is highly critical with regard to formalization, for a formalistic account of language? Mannoury himself suggests a formalization, first of the terminology of psychology, and he explicitly intends this formalization to "fill the gap, at least partially", between psychological and physical (or physiological) terminology—and that certainly is a typical task of significs, in complete agreement with the idea that there are multiple levels of language and with the anti-reductive program of integrating the various extreme positions into one over-arching theory.

His formalization can be presented in very concrete form. Obviously, he made use of a Meccano-construction kit (Mannoury had children) in order to build models for the elementary structures of psychical acts, thus reifying the "building blocks" he spoke about, and he presents these models in his lectures (the illustrations in Figures 1 and 2 come from Mannoury, 1934, p. 302).²³

These models are based on the idea that the elementary facts of psychical life consist in two kinds of excitations or drives, namely those of seeking pleasant and of avoiding unpleasant or undesirable circumstances. These can be considered as two poles generating an activity line of associations that can then be integrated into a larger, two-dimensionally extended network. These two-dimensional representations are again to be understood as a projection of a three-dimensional network, and it is this three-dimensional structure that Mannoury presents in his lectures via a Meccano-based model. The models indeed display, in their tendency towards abstraction and symbolic representation, important aspects of formalization, and it is worth noting that one can apply to them purely mathematical techniques such as projecting them onto different planes.

²³"Meccano" construction sets—originally called "Mechanics Made Easy", and thus incorporating an interesting element of popularizing science—were produced since the very first years of the 20th century; the name was adopted in 1908; cf. Love and Gamble (1986); Bowler (2009).

The figures from Mannoury 1934 cannot be reproduced in this online version for reasons of copyright. For the figures, see Mannoury, G. (1934). Die signifischen Grundlagen der Mathematik. Erkenntnis, 4:288–309, 317–345, at p. 302.

FIGURE 1.

Mannoury is quick to stress the restrictions of this model; it still is a "dürftiges Formalisierungssystem" (Mannoury, 1934, p. 306), a rather impoverished system of formalization that requires "further differentiation and filling in" in order to fit our ordinary language. But what, then, can be the function of this type of formalization? It is viewed as a tool that can, for instance, point towards "missing links"—this evolutionary jargon is quoted in English in the German text—, but can never serve as an ultimate foundation, "als eine Art Anfangspunkt" for a "Begriffslehre", a theory of concepts (Mannoury 1934, p. 307), from which a theory of concepts might be derived in a logical way. The foundations of language cannot rest on "dead formulae" but must be based upon the "living facticity of the concepts" ("die lebende Tatsächlichkeit der Begriffe selbst"), a phrase that with its phenomenological and humanities-based ring again evokes ideas from highly heterogeneous strands in philosophy.

Paul Ziche

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FIGURE 2.

5 Summary: multiple discoveries and dis-integration

Although in many respects related to the paradigm case for a multiple discovery, energy conservation, the multifarious claims regarding the discovery of logic follow a fundamentally different script. Whereas it becomes possible to integrate the various discoveries of energy conservation into one overarching framework (rooted in basic natural science with generally accepted

methodological standards), this becomes impossible in the case of the alternative discoveries of logic: scientists that—from today's perspective, at least—not only appear to work in very different fields employing incommensurable methodologies but also to exemplify completely different forms of rationality, lay claim, equally, to be the discoverers of modern logic.

The conclusions one can draw are far more concrete than in the case of multiples. For the latter, the theoretical background still is in a strangely embryonic state, given the attention devoted to this phenomenon by classical authors such as Merton and Kuhn. Rather few recent approaches to this problem exist, and Merton and Kuhn themselves at best touch upon some central issues. The seemingly overwhelmingly documented fact of multiple discoveries in science lends itself to theories about the psychology of discovery, the role of genius in science, or to a phrasing in rather vague categories such as that a discovery "lies in the air" or that the "time is ripe" for such a discovery (De Solla Price, 1963, p. 66–67; Kuhn, 1959, p. 70).²⁴

The scenarios of alternative discoveries do not require the problematical assumption that the various protagonists acted "in complete ignorance of each other's work" (Kuhn, 1959, p. 70). Rather, they point explicitly towards the importance of studying the interplay between distance and *unity* involved in the various alternatives, and to address the forms of distance separating the individual discoveries. Alternative claims frequently lead to a future *segregation* of the diverse fields, and in this process bring the relevant criteria for exhibiting unity versus diversity between fields into sharper focus.

The case studies presented here show that some of the most crucial concepts of modern logic, and of a philosophy that takes modern logic as its model, can be brought to bear on a surprisingly broad set of positions. Issues such as formalization and the problem of "Scheinprobleme" can be treated in both a boldly harmonizing way, embedding mathematically inspired forms of logic into a yet more comprehensive context, and in a fashion that leads to an increasingly more precise, and more thorough demarcation between scientific and philosophical fields.

The way in which comprehensiveness was to be achieved can itself be used as a criterion for giving precise contours to the various schools of thought co-existing around 1900. Two examples from the context that has been discussed here may be adduced. The writer Frederik van Eeden (who would make a particularly interesting case study for the fusion of different forms of rationality, given his interest in parapsychology), begins his book *Redekunstige grondslag van verstandhouding* ("Logical foundations of agreement")—which has been compared in form and content, to some ex-

 $^{^{24}}$ Lamb and Easton (1984), in their comprehensive classification of forms of multiple discoveries equally presuppose that genuine cases of multiple discovery are easy to find.

tent, with Wittgenstein's *Tractatus*—on a note reminiscent of the Vienna Circle's manifesto: there is a "need [...] in many people, for certainty, clarity and logical connections". Towards this end, he embarks on a "dialectical" clarification of language wherein the "symbolical meaning" of each word should be strictly preserved (van Eeden, 1897, p. 12*). This modifies and broadens the ideal of clarity considerably. What is more, the same holds for his view of logic: "the harmony of the parts, logical connection, the law of formulated thoughts that one calls logic, can only be reached in the case of a simple structure", beyond that we have to sacrifice the "harmony of living things" and the "harmony of reason" (van Eeden, 1897, p. 12*).

The dialectics of differentiation and integration become particularly clear in the division of labour suggested by the Amsterdam logician Evert W. Beth in a discussion of the meaning of significs for logic: "logical methods are adequate with regard to mathematical reasoning"; by contrast, the application of significal methods "will be efficient in a study of propaganda" (Beth, 1948, p. 84). But these two attitudes need not conflict: "both of them aim at clarifying our terminology, at improving our means of understanding". Plus, "in intermediate domains—in experimental science, in historiography, in literary criticism—, an intermediate point of view and a simultaneous application of both logical and significal methods will be most appropriate. Logical empiricism, for instance, may be considered as an intermediate position adapted to the peculiar nature of experimental science." What makes this bold harmonization particularly interesting is the juxtaposition of experimental science with the prototypical humanities; Beth, evidently, does not see a privileged link between mathematics, logic and experimental science, nor does he take experimental science, qua its ideal of rationality, to stand any closer to logic than to the humanities.²⁵

It also becomes obvious where the fault lines run along which these demarcation processes take place. Core concepts such as "science" and "formalization" are by no means clearly defined. Witness the juxtaposition of "experimental science" and the humanities in Beth and comparable statements in the writings of Ostwald. Consequently, such concepts cannot serve as guidelines along which to structure the development of logic, being themselves subject to revision in these processes. Mannoury tackles these issues by situating logic in a broad theoretical and cultural context and, thereby, realizes the significs' demand for integration of affective and logical dimensions. His inaugural lecture entitled *On the social meaning of the mathematical form of thinking* first restricts the role of formalization (Mannoury, 1917). He argues that formalization in mathematics shares its function with language in general: the ultimate goal lies in making thoughts and ideas

 $^{^{25}\}mathrm{Cf.}$ also the strong interest in the history of logic that is to be found in, e.g., Beth (1944a,b, 1946).

available as fixed points of reference. Then, significantly, he goes a step further and contrasts two conceptions of mathematics. On the one hand, mathematics can be seen as "gevoelloos, is onwezenlijk, [...] dood" ("without emotions, not essential, dead") adding, pensively: "En toch, en toch..." ("But still, but still [...]"). On the other hand, he stresses that contemporary debates on the foundations of mathematics were directly linked to deeper needs. These statements retrace the interpenetration of various lines of traditions: Beth extolls discoveries that stood at the cradle of modern logic (multidimensional geometry, transfinite numbers, point sets) in a hymnal, metaphysically charged language: these are the "sources... from which flows forth the clear certainty that enlightens the path of mathematics" and "sparks of light kindled by the last century in the field of mathematics".

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