SS 2026 Prof. Dr. Stefan Schwede
Dr. Tobias Lenz

Graduate Seminar on Advanced Topology (S4D4)

Compact Transformation Groups

Tuesdays, 14:15, SR 1.007

In this seminar we will study actions of compact topological groups in various contexts, which will bring together ideas from representation theory, differential geometry, algebraic topology, and commutative algebra.

Our main reference will be Bredon's book [Bre72], but you will sometimes want to consult additional sources for background material that Bredon does not treat in detail.

Prerequisites. We assume that everyone is familiar with the contents of the courses Topology I and II, in particular CW-complexes, manifolds, and Serre fibrations. For some of the talks, basic knowledge of differential geometry, representation theory, or algebraic geometry can be helpful.

Note. In many cases, the references contain more material than you will be able to present in your talk, and you should decide on which parts you want to focus (apart from the results explicitly mentioned in the talk description below, of course). As a rule of thumb, you should always try to present the proof of at least one non-trivial result in detail.

In the case of multipart talks, it is your own responsibility to get in touch with the other speaker to discuss which topics they are going to present in their talk (or conversely, which results they will need).

You should moreover meet with the assistant (Tobias Lenz, lenz@math.uni-bonn.de) at least two weeks before your talk to go through the material you want to present and to discuss any questions you might have.

Talks

1. **Topological groups** (M. Ramadan) .... April 14
Introduce topological groups, establish their elementary properties [Bre72, §0.1], and discuss various examples including classical matrix groups [Bre72, §0.2]. Give an overview of the proof of existence and uniqueness of the Haar integral for compact groups [Pon39, §25].

2. **Representation theory of compact groups** (D. Beligiannis) .... April 21
Introduce representations of compact groups, discuss examples, and prove the existence of invariant scalar products as well as complete reducibility of finite-dimensional representations [Bre72, §0.3]. Discuss a selection of the results from [Bre72, §0.4], in particular the characterization of matrix groups in terms of small subgroups.

3. **Lie groups** (S. Lange) .... April 28
Define Lie groups, introducing the requisite concepts from differential topology along the way. Build on the results of the previous talk to show that all compact Lie groups are matrix groups and that their orbits embed into representations. Give an overview of the classification of abelian connected Lie groups as well as the structure theory of compact Lie groups [Bre72, §§0.5–6].

4. **Transformation groups** (A. Sanchez Flores) .... May 5
Introduce actions of (compact) groups on topological spaces, discuss examples, and establish basic properties, in particular the equivariant Tietze extension theorem and the classification of transitive G-spaces [Bre72, §§I.1–4]. Discuss the basic constructions from [Bre72, §I.6].

5. **Principal bundles I** (J. Carvajal Noguera) .... May 12
Introduce principal bundles as in [Bre72, §II.1] and discuss the equivalence to the definition from [tD87, §I.8]. State [Bre72, Theorem II.5.8] (which will be proven in the next talk). Prove the homotopy classification of principal bundles over CW-complexes, see e.g. [Mit01, §8] and [tD08, §14.4]. If you have time left, discuss the connection to vector bundles and topological K-theory.

6. **Principal bundles II** (D. Arnold) . . . . May 19
Introduce tubes and slices, prove the existence of tubes for actions of compact Lie groups on completely regular spaces, and discuss the applications of this theorem to principal fibrations [Bre72, §II.4–5]. If you have time left, discuss equivariant principal bundles [tD87, §I.8].

7. **Smooth actions I** (I. Dankert) . . . . June 2
Introduce smooth actions of compact Lie groups on smooth manifolds, discuss examples, and prove the existence of invariant Riemannian metrics and invariant tubular neighborhoods [Bre72, §§VI.1–2]. This will require some Riemannian geometry, and you should give an overview of the necessary concepts and tools.

8. **Smooth actions II** (U. Gluck) . . . . June 9
Prove that smooth G-manifolds admit equivariant embeddings into representations and that equivariant continuous maps can be approximated equivariantly by smooth maps [Bre72, §VI.4]. If you have time left, discuss local smoothness and examples of actions of compact Lie groups on $\mathbb{R}^n$ that are not locally smooth [Bre72, §I.7, §IV.1]

9. **The Burnside ring I** (S. Pala) . . . . June 16
Introduce the Burnside ring $\mathbb{A}(G)$ of a compact Lie group G and discuss its relation to the universal Euler characteristic [tD79, §§5.2–5].

10. **The Burnside ring II** (C. Friedrich) . . . . June 23
Give an overview of the computation of the spectrum $\operatorname{Spec}\mathbb{A}(G)$ of the Burnside ring of a compact Lie group G [tD79, §§5.6–7], introducing the relevant definitions and results from elementary algebraic geometry as needed.

References

[Bre72] Glen E. Bredon, *Introduction to compact transformation groups*, Pure Appl. Math., vol. 46, Academic Press, New York, 1972.

[Mit01] Stephen A. Mitchell, *Notes on principal bundles and classifying spaces*, available at `https://sites.math.washington.edu/~mitchell/Notes/prin.pdf`, 2001.

[Pon39] L. Pontrjagin, *Topological groups*, Princeton Math. Ser., vol. 2, Princeton University Press, Princeton, 1939.

[tD79] Tammo tom Dieck, *Transformation groups and representation theory*, Lect. Notes Math., vol. 766, Springer, 1979.

[tD87] ———, *Transformation groups*, De Gruyter Stud. Math., vol. 8, De Gruyter, Berlin, 1987.

[tD08] ———, *Algebraic topology*, EMS Textb. Math., Zürich: European Mathematical Society (EMS), 2008.