Exercises to „Algebraic Geometry II“, 3

Exercise 1 (4 points). Let $0 \to \mathcal{A}^* \to \mathcal{B}^* \to \mathcal{C}^* \to 0$ be a short exact sequence of cochain complexes of sheaves abelian groups on a topological space X. Show the exactness of the long cohomology sequence formed with the connecting morphisms constructed in exercise 4 of the previous sheet.

Recall that on $\mathrm{Proj}(R.)$ there are quasi-coherent sheaves of modules $\mathcal{O}(k)$ such that $\mathcal{O}(k)_{\mathfrak{p}} \cong (R_{\mathfrak{p}})_k$ for any $\mathfrak{p} \in \mathrm{Proj}(R.)$. If the augmentation ideal is generated by R_1, these are line bundles. In particular, this is the case for

$$\mathbb{P}^n_A = \mathrm{Proj} A[X_0, \dots, X_n].$$

Exercise 2 (6 points). For arbitrary rings A and integers k, calculate $H^*\big(\mathbb{P}^1_A, \mathcal{O}(k)\big)$!

Exercise 3 (3 points). Show that any (possibly infinite) coproduct of quasicoherent sheaves of modules on a prescheme is quasi-coherent.

Exercise 4 (2 points). Decide whether the same holds for arbitrary products of quasi-coherent sheaves of modules on Noetherian schemes.

Recall that a covering $X = \bigcup_{i \in I} U_i$ of a topological space X is called *locally finite* if every $x \in X$ has a neighbourhood V intersecting only finitely many U_i. X is called *paracompact* if every open covering of X has a locally finite refinement.

Exercise 5 (5 points). Explain how to generalize the proof of Proposition 1.2.2. and Theorem 1 of the lecture to schemes whose underlying topological space is paracompact.

Solutions should be submitted Thursday, May 17, in the exercises.