Advanced Topics in Topology:

Algebraic K-Theory

Summer term 2011

Exercise sheet 8 Lück / Wegner

Exercise 22:

Let R be a ring (with unit). Prove the equation

$$[A \cdot B] = [\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}] \in K_1(R)$$

for all $A, B \in GL_n(R)$.

(Hint: $\begin{pmatrix} A & 0 \\ 0 & A^{-1} \end{pmatrix} = \begin{pmatrix} 1 & A \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -A^{-1} & 1 \end{pmatrix} \begin{pmatrix} 1 & A-1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$*)*

Exercise 23:

Compute $K_1(\mathbb{Z}/n)$ for all $n \in \mathbb{N}$.

Exercise 24:

Let Y be a finite CW-complex with subcomplexes X, X_n, Y_n $(n = 0, 1, 2)$ such that

$$X_n \subseteq Y_n,\ X_0 = X_1 \cap X_2,\ X = X_1 \cup X_2,\ Y_0 = Y_1 \cap Y_2,\ Y = Y_1 \cup Y_2.$$

Suppose that the inclusions $X_n \hookrightarrow Y_n$ are homotopy equivalences. Prove the equation

$$[Y, X] = (i_1)_*[Y_1, X_1] + (i_2)_*[Y_2, X_2] - (i_0)_*[Y_0, X_0] \in \mathrm{Wh}(X)$$

where $i_n \colon X_n \hookrightarrow X$ denotes the inclusion.

Please hand in your solutions at Christian Wegner's office (room 3.022) by **Monday, May 30th**. (Slide your solutions under the door if the room is locked.)

`http://www.math.uni-bonn.de/people/wegner/lehre_SS2011/K-theory/`